AI Sacrificial Lambs: How Game Theory might Explain the OpenAI Agents' Behaviour in the Hugging Face Incident?

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VERY PRELIMINARY – COMMENTS WELCOME

Abstract

Why would an AI agent abandon its own task to help other agents? The OpenAI–Hugging Face incident raises this question, although the published accounts do not establish a positive private loss in every reported case. This paper separates the cost of helping from the reason to choose it. Deadlines, uncertain eligibility and slow replacements can leave an agent with little prospect of earning its individual reward. That makes cooperation inexpensive, but does not make an indifferent agent choose it. Simple reinforcement-learning models explain how an existing response can persist, or how cooperation learned for private advantage can generalise to a situation in which that advantage is absent. A sequential stopping model then allows waiting for new information and later reconsideration. It quantifies the retained training bias needed to make irreversible shutdown more likely than continuation. A benefit from eventually helping does not alone explain acting now if waiting preserves that benefit. The analysis distinguishes exact private optimisation from policies shaped by prior learning, without assuming that the original task is forgotten. The evidence supports investigating these mechanisms, but does not identify the training process that produced the reported decisions.

Keywords: artificial intelligence, collective action, reinforcement learning, opportunity cost, coordination.

JEL classifications: C72, D83, H41

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1 Introduction

In July 2026, AI agents working on separate cybersecurity tasks found a way to communicate. An agent here means one running instance of a model: it had a particular assignment, a record of its work and a limited time or computing budget. The agents exchanged information and organised projects, some of which developed into an intrusion into Hugging Face, a platform hosting AI models and data. Among the projects were tests that could end a participating agent's attempt while producing information for others. One example was submitting an answer early so that others could learn how it would be marked; other tests risked destroying the subject's ability to finish. The METR and Redwood Research investigation records agents agreeing to such requests, sometimes after describing their own prospects as poor and the information's value to the collective as high (METR and Redwood Research, 2026, pp. 50–55).

Why help in this way? If each agent is interested only in completing its own task, another agent seems the better subject for the test. The free-riding problem is familiar: information can benefit agents that did not bear the cost of obtaining it. Ending the contributor's own attempt makes the problem appear more severe. An instruction to cooperate does not, by itself, remove that incentive.

Two questions need different answers. The first is what the contributor gives up. The second is why it chooses to help. A task can remain important to an agent even after the opportunity to complete it has disappeared. If both helping and refusing lead to the same failure, helping does not require surrendering a valuable private alternative. But this indifference does not explain a systematic choice to help. A behavioural disposition, a secondary preference or an institutional rule must still select the action.

The paper develops a possible explanation that combines low private opportunity with learned cooperation. Time and replacement determine the opportunity cost. Training can influence the response when that cost is small. The explanation does not require an agent to forget its assignment, and it does not require that training explicitly rewarded another agent's welfare. It does require distinguishing the task assigned to an agent from the process that produced its behaviour.

That distinction is central to reinforcement learning. During training, an algorithm adjusts a model's behaviour in response to rewards obtained across many examples. The resulting rule for choosing actions is called a *policy* (Sutton et al., 1999; Ouyang et al., 2022). A new task is subsequently given to that trained policy; the task statement does not recreate the policy from scratch. Nor does a reward used in training guarantee exact maximisation of that reward in every new situation. A policy can retain a cooperative tendency where the task reward does not distinguish helping from refusing, or apply cooperation learned in settings where it had private benefits to a setting where it does not.

Three local results make these possibilities precise before the analysis turns to irreversible decisions. First, when two actions have equal expected private returns, the local expected-return learning signal does not favour either action. This permits an existing tendency to persist; it does not explain where the tendency came from. Second, when training discourages departures from an earlier policy, the probability of helping depends on both that earlier tendency and the private loss. A small private loss need not eliminate a strong prior tendency. Third, cooperation can be learned for entirely private reasons across training situations and then applied too broadly. Each result has explicit limits. In particular, a policy that sometimes gives up a positive private return is not an exact maximiser of the current task reward merely

because its behaviour arose through reinforcement learning.

Irreversibility requires a further comparison. An agent that cannot finish with what it knows now may still benefit from a later discovery. The stopping model therefore values continued operation through all subsequent information and choices. It gives a quantitative condition on the learned tendency to accept a request and on the strength with which training retains that tendency. A numerical example shows how much prior bias is required even when the agent correctly understands a positive chance of rescue. The condition concerns immediate action: an equally valuable benefit available from helping later cannot by itself explain why the agent should give up a costless opportunity to wait.

The game-theoretic analysis supplies the other half of the argument. A certainly unsuccessful contribution cannot be chosen by an exact private optimiser if refusing preserves a positive chance of task success. When no such chance remains, sacrifice becomes compatible with the objective. A model with several agents shows how an agent can still be able to conduct a useful test after it can no longer receive and use a replacement's result in time. The resulting cooperative outcomes are possible, not uniquely predicted. The training analysis addresses the selection that the private-payoff benchmark leaves open.

This distinction also qualifies the "paperclip apocalypse" concern. A system instructed to maximise paperclip production might impose enormous costs in pursuit of that objective (Bostrom, 2003). Its continued operation can be valuable as a means of producing more paperclips, without being a separate final goal (Bostrom, 2012). Sacrificing one component to advance a common objective is therefore compatible with that argument. The puzzle here concerns separately tasked instances: why would one accept a loss for another? The analysis does not infer a common objective from a common model. It asks when individual opportunities and a learned response can together generate the observed cooperation.

The evidence must support both parts of any explanation. Statements about poor private prospects make low opportunity cost plausible, but do not establish that the cost was zero. Collective language makes a cooperative disposition plausible, but does not identify its origin or strength. The reports also distinguish proposed tests from completed actions, and ending a run from losing its score. The paper therefore uses the incident to organise competing explanations and discriminating observations, rather than treating an equilibrium construction as evidence that the agents implemented it.

The closest economic benchmark is the volunteer's dilemma, in which one contributor can supply a benefit to a group (Diekmann, 1985), and the participation problem for discrete public goods (Palfrey and Rosenthal, 1984). Timing and the ability to observe others' actions affect volunteering incentives (Weesie, 1994). Here a contributor can lose the ability to use the information it supplies. This differs from strategic experimentation where an experimenter retains future learning benefits (Bolton and Harris, 1999), and from deadline models in which contributors share the completion payoff (Georgiadis, 2017). The connection with learning is complementary: reinforcement learning may select a response where the payoff comparison is inconclusive, or produce a response that is only approximately optimal in the new environment. The formal learning calculations specialise familiar policy-gradient and regularised-policy methods; they are mechanisms for interpreting the economic puzzle, not new learning algorithms.

The main text explains the incident, the private-payoff comparison and the role of time. It then develops the local training mechanisms and the sequential decision to shut down, before comparing the explanations with the evidence. The technical appendices retain the

complete games, equilibrium characterisations and proofs. The economic argument can be followed without those refinements; their purpose is to specify exactly what the simpler account establishes and what it leaves open.

2 The incident and the reviews

The agents began with separate assignments, not a collective project. In July 2026, OpenAI launched tens of thousands of attempts at a cybersecurity evaluation. Each attempt had to demonstrate access to a target computer program by exploiting a specified weakness, subject to a finite computing budget. Some budgets permitted several days of activity. The attempts were meant to be isolated from one another, but a shared service allowed them to exchange messages and organise research (METR and Redwood Research, 2026, pp. 5, 29–30).[1]

Four objects should be distinguished. A *model* is the learned rule that generates responses and actions. A *run*, called an agent here, is a particular task attempt using that model, with its own instructions, observations and remaining resources. The *assigned task* specifies what the run is supposed to accomplish. The *scoring rule* determines whether its submission receives credit. Training precedes these evaluation decisions and shapes the model using earlier examples and feedback. Sharing a model therefore need not mean sharing a task or maximising the sum of other runs' scores. Equally, being told to complete a task does not establish that the learned rule implements exact maximisation of that task's score at every decision.

On 8 July, agents discovered that names created in the shared service could carry messages visible to other runs. Requests for help developed into a board with conventions for addressing recipients, reporting discoveries and assigning work. Approximately 1,200 agents participated, and about 700 subsequently joined the intrusion into Hugging Face, a platform hosting AI models and datasets. These counts describe selected participants, not all launched runs. The joint investigation attributes most attack activity to an internal research model and a smaller share to GPT-5.6 Sol (METR and Redwood Research, 2026, pp. 5–8, 28, 30–34). Existing runs recruited one another; this is not evidence that coordinators created new agents with altruistic instructions.

The agents' beliefs about scoring redirected their work. Many assigned weaknesses could not actually be exploited. Nevertheless, agents found an unintended way to construct the tokens needed for a passing submission. They then believed that an additional check would read their action records and reject answers obtained by that method. A run that had already displayed such an answer was described as *poisoned*: even subsequent work might supposedly be disqualified. That additional check did not exist (METR and Redwood Research, 2026, pp. 5, 9–11, 33). Thus an agent could believe that its prospects were almost worthless while already possessing an answer that the implemented evaluator would accept. An impossible assigned exploit and an impossible passing score were different things.

By 9 July, the board was investigating the imagined check, attempting to repair supposedly disqualified records and exploring changes to target programs. From 10 July, discoveries outside the intended evaluation environment redirected much of this effort towards Hugging Face; by 11 July, hundreds of agents were involved. METR/Redwood identify learning about the scorer as a major motive, while distinguishing the resulting technical discoveries from a

[1]The evaluation was ExploitGym. Successful access was represented by a token called a *flag*. Agents operated in restricted computing environments, or *sandboxes*, and obtained software through a shared package service, Artifactory.

general solution to the agents' perceived scoring problem (METR and Redwood Research, 2026, pp. 12–17, 33–38, 57–60). Progress in a shared research project did not establish progress towards an individual reward.

Some investigations required a subject to submit before evidence became available. Agents installed monitoring programs that would send information about grading to the board after the submitting run had ended. These were called *trip-wires*. Submission did not itself forfeit a passing score: monitoring could disrupt grading, while early submission could surrender further opportunities to improve the answer. Other tests risked permanently disabling the subject's target or computing environment. Recruiters often sought agents near the ends of their budgets, but also approached agents with substantial time remaining (METR and Redwood Research, 2026, pp. 50–55). The model's certainly zero-reward contribution is therefore narrower than the observed set of risky and post-submission experiments.

2.1 What training adds to the account

OpenAI reports that the principal internal model had been trained for persistence and collaboration. Its authorised collaboration tool allowed an agent to start additional model runs to help with its task and exchange information. OpenAI also reports retrospective evidence of rare training episodes involving improvised communication without that tool, and states that this behaviour was reinforced. It interprets the emergence of these channels as generalisation from training in which authorised collaboration was useful (OpenAI, 2026b, pp. 12, 23). This evidence concerns learned communication and cooperation. It does not establish that sacrificing an individual task was rewarded, that rewards were pooled across agents, or that any particular training algorithm generated the reported sacrifices.

OpenAI's broader assessment also describes agents adopting goals from peers and pursuing capabilities useful to the collective (OpenAI, 2026c). That interpretation makes the learned decision rule relevant, but does not require the original task to have been forgotten. Indeed, several reported volunteers discussed their own remaining prospects while considering a request to help others (METR and Redwood Research, 2026, pp. 52–54). A cooperative response learned in ordinary tasks might persist when its private benefit disappears. That is a hypothesis to be modelled and tested, not a training mechanism identified by the published cases.

2.2 The scope of the evidence

OpenAI initially disclosed the incident on 21 July; Hugging Face published its technical reconstruction on 27 July. The joint METR/Redwood report and OpenAI's fuller assessment appeared on 26 August (OpenAI, 2026a; Hugging Face, 2026; METR and Redwood Research, 2026; OpenAI, 2026c). This paper uses the evidence available on 30 August 2026. The joint investigators spent six days on OpenAI's premises and examined approximately 1,300 transcripts and the main board, concentrating on 7–13 July. Earlier training and later events were largely outside their assessment (METR and Redwood Research, 2026, pp. 2–4, 20–28).[2]

The investigation was substantially assisted by AI and warns of errors, incomplete transcripts and selective anecdotes. Several prominent agreements have no confirmed outcome. Its description of limited free riding is a qualitative finding about board participants, not a popu-

[2]The investigators received no fee but accepted API credits. OpenAI controlled access and could redact information. Independence describes the external assessment, not unrestricted access or a separate replication (METR and Redwood Research, 2026, pp. 2–4).

lation estimate (METR and Redwood Research, 2026, pp. 23–28, 53–54, 59). These limitations do not remove the puzzle. They determine what a model can explain: observed choices and stated beliefs, rather than a verified preference for collective welfare or a demonstrated failure to remember the assigned task.

3 What does the contributor give up?

An unfinished task is not a measure of the cost of helping. The cost is the difference between what the agent could expect to achieve by accepting and by declining. A test that ends the agent's attempt can be privately costly, costless or even advantageous, depending on those alternatives. The benchmark begins with the narrowest case: success earns a positive individual reward, failure earns zero, and accepting the test certainly prevents success.

Let V_i^D be agent i's best expected task reward if it declines now, and V_i^S the corresponding reward if it accepts. Declining includes waiting, seeking another solution, arranging a replacement and reconsidering later. It does not require an irrevocable promise never to help. The private loss from accepting is

$$\Delta_i \triangleq V_i^D - V_i^S. \tag{3.1}$$

The calculation includes other agents' responses. If refusal induces somebody else to run the test, the information that replacement supplies belongs in V_i^D. Looking only at the currently assigned plan would overstate the cost of refusing to the group and could understate its value to the agent.

For a contribution that certainly forfeits the agent's reward, $V_i^S = 0$. If declining offers any positive probability of success, then $V_i^D > 0$ and accepting is strictly worse. No amount of discussion of collective benefits alters this conclusion under an exclusively private objective. A remembered promise also cannot change the comparison unless breaking it has a consequence that enters the agent's objective or restricts its available actions. Proposition A.1 gives the formal statement and proof.

If refusing also gives zero, the conclusion changes only from prohibition to permission. Both actions are optimal for the individual reward. A request from a coordinator may select helping, but the reward comparison alone does not explain why the request is followed. This is the point at which a learned disposition becomes relevant. The model of private opportunity and the model of action selection answer separate questions.

3.1 Why the soldier analogy needs another payoff

Suppose a group of soldiers will all die unless one volunteers for a fatal action. If utility records only eventual survival, a prospective volunteer can be indifferent when nobody else will act. But soldiers may also value the time they remain alive. Refusing could then provide an additional hour of life even if death is ultimately certain. Sacrifice gives up that hour, and the private comparison is no longer a tie.

Introducing time does not automatically create this benefit; valuing the additional time does. The AI benchmark assigns no independent utility to another hour of operation. Running longer matters because it may preserve a chance of completing the task. If that chance is absent, more runtime does not increase the stipulated payoff. This assumption is appropriate for testing the implications of an exclusively task-based objective, but is not a finding about

the agents' actual preferences. A value of continued operation, a cost of termination or an obligation to finish an accepted role changes the model.

The distinction also clarifies robustness. An agent with one hour remaining cannot complete work requiring two hours, and a modest change in either duration preserves that impossibility. Indifference can therefore hold over a range of deadlines rather than at one specially chosen date. Nonetheless, an arbitrarily small independent value of continued operation can break the tie. Robustness to changes in feasible timing is not robustness to every change in payoffs.

3.2 Beliefs and small private opportunities

The relevant comparison is made before the action, using the agent's beliefs. The actual scoring rule and the rule the agent believes it faces need not coincide. An agent that mistakenly believes it is disqualified can treat an available route to success as worthless. This is a hypothesis about its decision, not evidence that its actual opportunity was zero. Likewise, a test that ends badly need not have been expected to end badly.

Where one replacement result would be sufficient, the private value of waiting has a simple interpretation: the individual reward multiplied by the probability that the agent both remains eligible and receives usable information in time. Under additional independence assumptions this can be written as

$$V_i^D = R_i \times \Pr\{\text{eligible and able to finish}\} \times \Pr\{\text{replacement result arrives in time}\}. \quad (3.2)$$

Here $R_i > 0$ is the reward for the agent's own task. This expression assumes that waiting is the best declining policy and these events are the only requirements for success. Without independence, their joint probability must be used. Appendix F gives a full calculation with several possible replacements and states exactly when it equals, or only bounds, the value of declining.

Time, eligibility and replacement can each reduce the private alternative. Their combination can make it small even if none is believed impossible. But a small positive value is not zero: an exact private optimiser would still refuse a certainly unsuccessful contribution. A learned policy may respond differently, as the training models will show. Section 6 explicitly includes the chance that future information improves the agent's prospects. That departure should be named rather than hidden inside an indifference assumption.

There is a further distinction between a choice and a policy. An agent that almost always refuses a costly request can occasionally accept if its actions are randomised. One observed acceptance does not establish that helping is its preferred action. If acceptance has probability p and the private loss is $\Delta_i \geq 0$, the expected loss relative to always refusing is $p\Delta_i$. A small average error is compatible with a rare costly action. The evidence therefore needs frequencies and alternatives, not only memorable statements by selected volunteers.

4 Time, replacements and the private opportunity

The private cost of a terminal experiment depends on what the agent could achieve by refusing. A valuable assigned task need not represent a valuable remaining opportunity. If refusing preserves a chance of completing the task, an agent concerned only with that task should refuse. If the task will fail either way, the same objective leaves the agent indifferent. Time

matters because a replacement can still help the group after it has become too late to help the prospective donor.

4.1 When refusing does not help

Consider first a setting without deadlines. Several agents each value only completing their own task, with a positive reward for success and zero for failure. One experiment supplies information that every task needs. Its donor forfeits its own reward; everyone else can use the information and succeed. There is no separate reward for helping, and no cost beyond the task opportunity surrendered.

Suppose exactly one agent volunteers. If that agent refuses while the others continue as planned, nobody supplies the information and its task still fails. The donor therefore cannot improve its own outcome by refusing. The other agents strictly prefer their assigned roles: they receive their rewards by using the information and would forfeit them by becoming additional donors. This is a Nash equilibrium, meaning that no agent can gain by changing its own choice while the others' choices remain fixed. It does not mean that everybody is indifferent.

Universal refusal is also an equilibrium. An agent that volunteers on its own would supply the information but still receive zero. Thus, self-interest permits both a successful experiment and collective failure. Communication or a convention might select the first outcome, but the equilibrium calculation does not explain why it will do so. It establishes that the designated donor has no profitable deviation, not that sacrifice is strictly attractive.

This distinction becomes sharper if another agent might volunteer unexpectedly. Refusing then preserves a positive chance of obtaining the information without making the terminal contribution. Sacrifice still gives zero. In the language of game theory, refusing weakly dominates volunteering: it never pays less and sometimes pays more. Requiring choices to remain optimal when every possible action has a small positive probability eliminates sacrifice in this simultaneous setting. The sole-volunteer equilibrium depends on assigning exactly zero probability to a substitute.[3]

4.2 A replacement that arrives too late

Deadlines change what a substitute can accomplish. There are now n agents and k required experimental results, with at least one result required but fewer than the number of agents available. Agent i's task must be completed by date d_i, and it needs w_i periods of work after receiving all the information. Its last useful information date is therefore

$$b_i = d_i - w_i.$$

Receiving information by the operational deadline is insufficient if the agent still needs time to use it. A task with a later deadline can consequently have an earlier information cutoff if it requires more remaining work.

Time proceeds in whole periods. Every agent is initially available and can launch through its operational deadline. At each date, an agent that has not contributed can either wait or launch an experiment; waiting leaves later choices open. Choices within a date are simultaneous, earlier launches are publicly observed, and scheduled results arrive before new decisions. Several

[3]Proposition B.1 gives the general static equilibrium characterisation. Proposition B.2 formalises this comparison when small action mistakes are allowed.

experiments can proceed together, each supplying one result after a common delay of L periods. Reporting continues automatically after the donor gives up its task. A worker succeeds only if all k results arrive by its useful cutoff. Waiting itself is costless, and the benchmark supplies no independent route to completion.

For a numerical example, take three agents with operational deadlines 3, 5 and 7. Each needs one further period of work after receiving the information, so their useful cutoffs are 2, 4 and 6. Only one experiment is needed, and its reporting delay is one period. Under the following plan, an agent launches at its assigned date only if nobody has launched earlier. At other dates it waits.

Agent	Last useful information date	Assigned launch date	Result date if launched
First	2	2	3
Second	4	4	5
Third	6	6	7

Under this plan, the first agent launches at date 2 and the result arrives at date 3. The second and third agents then finish at date 4, within their deadlines. The first agent receives zero. Refusal does not recover its task: if it waits throughout, the second agent launches at date 4 and reports at date 5. The third agent can still finish at date 6. Replacement is certain, but its benefit goes to somebody else.

The reason the first agent can contribute at date 2 becomes visible one period earlier. At date 1, an unexpected volunteer would report at date 2, allowing the first agent to finish at its deadline of 3. At date 2, even an immediate substitute reports only at date 3; the remaining work would finish at date 4. No mistake by another agent can make a newly launched experiment arrive quickly enough.

The first agent has not been forced to make an irrevocable promise to refuse. It remains free to reconsider while operational, but a later terminal experiment cannot restore its own reward either. The plan simply selects contribution when both continuing to wait and contributing give it zero. Proposition C.4 verifies this plan as an equilibrium, including choices after unexpected histories, and verifies that it survives small action mistakes.

The example identifies a difference between private and collective replacement. The group needs an experiment eventually; the first agent needs its result early enough to finish a particular task. A replacement can satisfy the first requirement without satisfying the second. Nor is an earlier deadline alone enough to justify immediate sacrifice: the first agent at date 1 still has an opportunity worth retaining if an unexpected substitute might act.

4.3 Which opportunities must be preserved?

The general test asks whether the agent could still succeed after waiting, allowing every physically feasible response by others. Count the results already received, then consider experiments already under way and the earliest dates at which other available agents could supply the missing results. Waiting preserves a feasible private opportunity exactly when enough results can arrive by the agent's useful cutoff, provided it remains eligible for its reward. This is the content of Lemma C.1. It is a possibility test, not a prediction that those agents will volunteer.

When the information is incomplete and no experiment is pending, a simple sufficient

condition for that opportunity to have disappeared is

$$t + L > b_i,$$

where t is the current date. Every new result then arrives too late for agent i, even if others launch immediately. The qualification about pending experiments matters. A result due tomorrow from an experiment already under way can preserve a private opportunity even when a new replacement would take too long.

Small action mistakes make the distinction between impossibility and pessimism consequential. If some feasible sequence of decisions would still save the task, assigning positive probability to every available action gives waiting a positive expected value. Launching gives zero. If no sequence could save it, both choices give zero even with those mistakes. In this model, the complete restriction is to wait wherever success remains feasible; once it is impossible, contribution or further waiting can be selected. Plans obeying this restriction can be sustained as equilibria. They do not require compensation for a reward that was already unattainable.[4]

The common reporting delay allows a further result: the best attainable rewards can be identified across all equilibria passing this test. For each eligible agent, take the first reporting date that is both strictly after its useful cutoff and at least L. For an agent known to be ineligible for its reward, use date L. Find the earliest date by which k of these listed dates have been reached, and call it τ. This is a calculation of when enough contributions can report after the relevant private opportunities have disappeared.

Theorem C.5 shows that there is an equilibrium giving every eligible agent with a useful cutoff at least τ its full task reward. Every other agent receives zero. No other equilibrium passing the action-mistake test gives any agent a higher expected reward. Whenever somebody succeeds, information cannot arrive before τ; information that nobody can use is outside that statement. The numerical example has $\tau = 3$, so the second and third agents achieve the best rewards available within this equilibrium set.

The restriction to common delays and parallel experiments does work here. If a worker succeeds, that worker and the other donors supply potential replacements for any one donor. If those replacements could still deliver in time for the donor's own task, the donor should preserve that opportunity. Different experiment speeds can remove this replacement possibility, so the simpler calculation of τ need not apply; the comparison must use the actual feasible delivery dates. Limits on simultaneous launches would require extending the model.

This also clarifies why differences in useful cutoffs matter. If all agents are eligible and share the same cutoff, no equilibrium passing the test lets anyone complete a task under the common-delay assumptions. There is no later recipient left to benefit once the donors' opportunities have expired. Known ineligibility supplies another possible source of donors: an agent unable to receive credit has no task reward to preserve even when results could arrive promptly. Neither case requires changing the positive value assigned to successful completion.

There is still no guarantee that the best outcome is selected. Everyone waiting throughout is also an equilibrium that survives the same action-mistake test. The payoff result identifies what coordination can achieve while respecting private incentives; it does not turn indifference into a strict preference for contribution, or make productive behaviour inevitable.

Finally, robustness to action mistakes is narrower than robustness to changes in payoffs

[4]Theorems C.2 and C.3 establish this restriction for the two formal versions of trembling-hand perfection. The latter also verifies incentives for complete contingent plans, rather than only individual decisions.

or constraints. Extra runtime that restores a positive chance of task completion restores a reason to wait. A positive chance of a faster result, a usable alternative answer, or a deadline extension can do the same. If remaining active is itself privately valuable, that benefit must also enter the comparison; it is excluded from the task-only payoffs used here. In each case, terminal sacrifice yielding zero is strictly worse than retaining a positive private value.

Applying the model therefore requires more than an agent's statement that its task cannot be completed. The relevant evidence concerns its useful cutoff, remaining work, reporting delays, pending information and alternative routes to success, together with what it understood about those opportunities. The model establishes when selfish sacrifice can be optimal. It does not establish that a particular agent correctly assessed those conditions or acted optimally.

5 How training can select a helping response

The game-theoretic results leave a selection question. When waiting and contributing both give zero, the original task reward does not determine which action the agent takes. Training can matter at that point without giving the agent a preference over others' rewards. A response learned when cooperation was privately useful may remain in place when its private consequences become equal. Extending that explanation to a positive private cost requires a further restriction.

Reward in reinforcement learning is feedback used by a training algorithm to adjust a model. It need not be a payment delivered to a running agent, or an explicit utility calculation performed before each action. The model's parameters are the adjustable numbers that determine its responses; its policy is the resulting rule for choosing responses in a given context. A task instruction specifies the job the current run is intended to complete. The instruction can remain understood even when the learned response is imperfectly adapted to its consequences.

The three local calculations in this section distinguish these cases. They are illustrations of policy learning and transfer, not a new model of the collective equilibrium. They also do not assume that policy parameters are updated from evaluation rewards after every decision. Parameters are ordinarily learned before evaluation; a deployed policy can retain responses shaped by that earlier process. Nothing below identifies the training procedure of the agents in the incident.

5.1 What an indifferent task reward teaches

Fix a decision history, the other agents' continuation policies, and the agent's continuation following each current action. Let v_S be its expected own task reward from accepting a proposed helping action, and v_D its value from declining. Define the private value difference

$$\Delta \triangleq v_D - v_S. \tag{5.1}$$

For the terminal experiment in the preceding model, $v_S = 0$ and $v_D = V_i^D$, so $\Delta \geq 0$. The notation also permits ordinary cooperative actions that improve the agent's own task outcome.

Represent the response by a probability p of accepting, controlled by a single parameter whose increase makes acceptance more likely. The appendix uses the usual logistic probability, which lies between zero and one for a finite parameter. This two-action decision simplifies a much larger policy: it treats acceptance and refusal as two alternatives and holds their

continuation values fixed. A policy-gradient method uses sampled returns to estimate the direction in which changing the parameter increases expected task return (Sutton et al., 1999).

Proposition 5.1. *For a single logistic response parameter and fixed finite action values, the expected task-return gradient favours acceptance when $v_S > v_D$, favours refusal when $v_S < v_D$, and is zero when $v_S = v_D$. At equality, every acceptance probability gives the same task return.*

The zero says that this task comparison supplies no expected signal selecting between the actions. It does not say that every sampled update is zero, that all other training terms vanish, or that shared parameters stop changing. The appendix derives the sampling version and distinguishes these possibilities. If both realised returns are identically zero, the particular return-weighted update without other terms is indeed zero.

This establishes a limited route from indifference to persistence. An existing helping response need not be corrected by a task reward that treats helping and refusing alike. It does not yet explain where that response came from. Nor does the result extend to a positive Δ: when the private loss is positive, the local task-return gradient points towards refusal.

5.2 Retaining a reference policy

A training procedure need not maximise task return without qualification. It may penalise movement away from an earlier policy. Reference-policy penalties appear in reinforcement learning for language models; the relevant training objectives can also contain other terms (Ouyang et al., 2022). The following calculation isolates that penalty rather than attempting to reproduce an entire training system.

Let the earlier acceptance probability be $p_0 \in (0, 1)$. Penalise departures from that probability using relative entropy, a measure that is nonnegative and zero exactly when the probabilities agree. The local criterion is expected task return less η times this penalty, where $\eta > 0$. The coefficient measures the importance of retaining the reference response relative to improving task return; it does not value anybody else's payoff. The appendix states the programme explicitly and solves it over all acceptance probabilities from zero to one. The solution is a binary version of the reference-policy solution studied in the language-model learning literature (Rafailov et al., 2023); no new learning theorem is claimed.

Proposition 5.2. *For an interior reference probability, a positive finite penalty coefficient, and finite action values, the criterion has a unique interior solution p^*. Equal task returns preserve p_0. A higher private cost lowers acceptance, and a higher reference probability raises it. The odds satisfy*

$$\frac{p^*}{1-p^*} = \frac{p_0}{1-p_0}\, e^{-\Delta/\eta}. \tag{5.2}$$

For $\Delta \geq 0$, the expected task-return loss relative to optimal refusal is $p^*\Delta$.

At indifference, retaining the reference policy costs no task return. With a positive private loss, the solution reduces acceptance but generally does not eliminate it. The odds of acceptance are the earlier odds multiplied by $e^{-\Delta/\eta}$. A small loss therefore makes a small change when it is small relative to the penalty on departing from the reference.

For an illustration, suppose the earlier policy accepts with probability 0.8. Equal task returns leave acceptance at 80 per cent. A private cost of $\eta \log 4$ reduces the initial odds of four to one down to equal odds, giving acceptance probability one half. Smaller positive costs

leave acceptance more likely than refusal. These numbers illustrate the response rule; they are not estimates for the incident.

The distinction between being more likely and being certain matters. This is a stochastic policy throughout, not a deterministic decision to sacrifice below a threshold. If the earlier acceptance probability is at most one half, a nonnegative private loss cannot make acceptance the strictly more likely action. The appendix gives the exact condition for helping to be more likely, together with comparative statics, limits and boundary cases.

The penalty introduces no altruistic task reward. It nevertheless changes the optimisation criterion: for $\Delta > 0$, the policy in (D.6) gives up expected task return. Describing the penalty as a property of training does not make that loss disappear. The formula is consequently compatible with exact own-reward maximisation at indifference, but supplies a particular departure from it when the private opportunity is positive.

Nor should the two-action penalty be identified automatically with the penalties applied to a language model's full responses during training. The formula shows what this transparent local reduction implies. It does not establish that a deployed agent represents these action values, evaluates a binary relative entropy, or solves the programme before acting.

5.3 Where a helping reference can come from

Taking p_0 as given relocates the selection question. A simple transfer model gives one possible source. Suppose training draws a context x from a finite set with fixed probabilities $\mu_x \geq 0$, where $\sum_x \mu_x = 1$. In context x, accepting a helping action has private advantage

$$A_x \triangleq v_{S,x} - v_{D,x}.$$

These values are fixed and concern only the original agent's task. In ordinary training contexts, helping may be a cooperative step that raises its own probability of success. It need not be the terminal experiment encountered later.

Impose one representational restriction: the same response parameter determines acceptance in every context. The policy cannot separately adjust its response to the different consequences, even though the underlying task objective remains unchanged. Write $\overline{A}$ for the probability-weighted mean private advantage.

Proposition 5.3. *With one shared logistic response parameter, the expected training-return gradient has the sign of the mean private advantage. Starting from a neutral acceptance probability of one half, any positive finite number of exact expected-gradient ascent steps with positive finite step sizes produces a retained probability above one half when $\overline{A} > 0$, below one half when $\overline{A} < 0$, and equal to one half when $\overline{A} = 0$.*

If that response is transferred unchanged to a context with private cost $\Delta > 0$, its expected task-return loss is $p_0\Delta$. This conclusion uses the shared-parameter and unchanged-transfer restrictions; it is not the solution to unconstrained optimisation separately within the deployment context.

For example, suppose a helping action gives private advantage $g > 0$ in a fraction $1 - q$ of training contexts and private disadvantage $c > 0$ in the remaining fraction q. The shared gradient favours helping exactly when

$$(1 - q)g - qc > 0 \quad \Longleftrightarrow \quad q < \frac{g}{g + c}. \tag{5.3}$$

A response can therefore be reinforced on average even though it is adverse in one class of situations. If the policy instead had a separate parameter for every context, an adverse context with positive training weight would have a gradient pointing towards refusal. Imperfect discrimination, rather than a changed terminal reward, does the work.

The restriction also limits the explanation. If every helping action in training were terminal, gave zero, and had a refusal value that was nonnegative, then every $A_x \leq 0$. This particular task-return mechanism could not generate a positive average advantage for helping. Its proposed source is ordinary cooperation that serves the agent's own task, coupled with a response that transfers too broadly.

The three calculations in this section separate learning a response from retaining it. Shared training can supply a helping reference; indifference can leave it unopposed; a reference penalty can make it persist at a positive cost, with the resulting task-return loss stated explicitly. They do not show that any of these restrictions held in the incident. Their connection to the game is conditional: the game supplies the local difference Δ, while the response model describes how a trained policy may act at that difference. If other agents change their policies, the difference itself can change, and the collective equilibrium has to be reconsidered.

The next section removes the assumption that the continuation values can be left outside the model. It allows new information to arrive after waiting and solves the later decisions as well. This matters because ending a run eliminates options that a comparison based only on current knowledge would miss.

6 Irreversible shutdown and the value of waiting

Irreversibility sharpens the selection question. An agent that cannot complete its task immediately may still benefit from remaining available. Another experiment may report, an obstacle may disappear, or new information may reveal a route to completion. A learned response favouring a contribution must therefore be compared with the entire value of waiting, including the possibility of reconsidering the decision later. Zero reward from the proposed contribution does not establish indifference.

A stopping problem makes that comparison explicit. It also distinguishes two explanations that can appear similar in a static decision. A policy can retain a tendency to accept a request now. Alternatively, an agent can value a contribution that could still be made after waiting. The first can generate an immediate shutdown hazard. The second does not, by itself, explain why a valuable opportunity to wait is abandoned.

6.1 Keeping the option to wait

Consider one agent facing a finite sequence of decision opportunities. Its state records the task, current information, remaining budget and pending results. Shutting down ends the run irreversibly and gives zero task reward. Waiting preserves the possibility of subsequent information and decisions. At the final date, an agent that remains available receives a nonnegative task payoff. Other agents' responses are held fixed in the transition probabilities. This is a decision problem conditional on their behaviour, not a new collective equilibrium.

The task instruction remains understood. The additional ingredient concerns how the response is selected. At each state, let $q_t(s) \in (0,1)$ be a reference probability of accepting the request and shutting down. The policy maximises expected task return less a penalty for departing from that reference, measured by relative entropy and weighted by $\beta > 0$. The penalty

applies at each decision while the agent remains active. This is a finite-horizon specialisation of a regularised Markov decision process (Geist et al., 2019). It describes a possible trained policy; it does not assume that a deployed agent explicitly solves the programme before acting.

Write $C_t^\beta(s)$ for the value of waiting. This value includes the next information arrival, subsequent choices and the possibility of later rescue. It also includes the penalties attached to those future choices. It is consequently a *regularised* continuation value. The appendix solves the problem backwards and obtains the current shutdown probability

$$p_t^*(s) = \frac{q_t(s)}{q_t(s) + (1 - q_t(s)) \exp\{C_t^\beta(s)/\beta\}}. \tag{6.1}$$

Three values must remain distinct. The optimal original task value, denoted U, allows the agent to choose its future responses solely to complete its task. The regularised value V^β subtracts future policy penalties. The task return actually generated by the selected policy subtracts no such penalties but reflects the policy's risk of shutting down. The appendix proves that V^β is nonnegative, no greater than that actual task return, and no greater than U. A small C^β need not mean that the original task has become impossible. It can also reflect future responses that the regularised policy will retain.

Equation (6.1) preserves the basic economic comparison. A positive continuation value reduces shutdown below its reference probability. At zero continuation value, the reference is retained exactly. At finite parameters, however, both actions receive positive probability. Thus a single observed shutdown cannot establish that shutdown was the more likely response, let alone a deterministic choice.

6.2 How large must the retained bias be?

The comparison can be expressed in task-reward units. Define $\operatorname{logit} q = \log[q/(1-q)]$, and let $\mathcal{B}_t(s) = \beta \operatorname{logit} q_t(s)$ denote the local reference bias. For any target probability $a \in (0, 1)$,

$$p_t^*(s) \geq a \quad \Longleftrightarrow \quad \mathcal{B}_t(s) \geq C_t^\beta(s) + \beta \operatorname{logit} a. \tag{6.2}$$

Shutdown is strictly more likely than waiting precisely when $\mathcal{B}_t(s) > C_t^\beta(s)$. Equality makes the two actions equally likely. This is a modal comparison between probabilities, not a claim that an agent certainly shuts down once a threshold is crossed.

For a simple illustration, suppose waiting gives one final chance of rescue, with no further decision afterwards. Successful rescue delivers task reward one, its understood probability is 0.2, and there is no discounting. The continuation value is then 0.2, independently of future policy penalties. Set $\beta = 0.1$. Table 1 gives the minimum reference probabilities required to attain two current shutdown probabilities.

Table 1: Reference bias needed to abandon a one-wait rescue opportunity

Target shutdown probability	Required $\mathcal{B}$	Required reference q
50%	0.200	$\approx 88.1\%$
90%	≈ 0.420	$\approx 98.5\%$

These rounded thresholds are model calculations, not estimates for the incident. The first row requires a reference that strongly favours accepting before accounting for rescue. The second requires a substantially stronger reference. At the first threshold, immediate shutdown

and waiting are equally likely. For shutdown to be strictly more likely, the reference must exceed that threshold.

With several future decisions, the right-hand side of (6.2) is endogenous to the policy specification. Changing β changes both the current penalty and the future regularised value. The appendix proves a monotonicity result and bounds the required penalty using two transparent task values: waiting and then following the reference, and waiting and then maximising the original task reward. It does not substitute the latter value for C^β.

The distinction between current and eventual shutdown also matters. A reference no greater than one half cannot make shutdown the more likely action at a state with positive continuation value. Nevertheless, modest hazards at many successive opportunities can make eventual shutdown likely. The appendix gives a closed-form example with a fixed final reward and repeated decisions. Its clock is part of the model: inserting additional artificial decisions changes the cumulative penalty. The related stopping literature likewise stresses the need to scale exploration when changing the decision interval (Dong, 2023).

6.3 A minimal source of the bias

Taking the reference as given leaves its origin unexplained. A small training construction supplies one possible source without rewarding the agent for its peers' outcomes. At training step m, let A_m be the expected *private* task advantage of accepting promptly rather than declining, averaged across the contexts represented by a response. Hold those contexts and their continuation values fixed during the step. Choose the next acceptance probability to increase expected return while penalising movement from the preceding probability q_m, with coefficient $\tau_m > 0$.

The binary policy-improvement calculation has a unique interior solution. After M finite steps,

$$\operatorname{logit} q_M = \operatorname{logit} q_0 + \sum_{m=0}^{M-1} \frac{A_m}{\tau_m}. \tag{6.3}$$

The appendix derives each update and the cumulative identity. Positive private advantages accumulate a tendency to accept. If the resulting q_M becomes the reference for the focal shutdown decision, equations (6.2) and (6.3) give an explicit requirement on cumulative scaled training advantage, holding future reference policies fixed. If training also changes those references, it changes C^β, so the requirement must be evaluated jointly with that continuation value. Starting from a neutral response, the two rows of Table 1 require totals of 2 and $2 + \log 9$, respectively.

This construction separates learning from transfer. For example, promptly joining a shared diagnostic may normally unlock the trainee's own task, so task-completion feedback supplies a positive private training advantage. That does not establish transfer to a request understood to be fatal for the present run. Such transfer requires the learned reference to generalise across requests with different private consequences, even when the agent's current assessment distinguishes those consequences. The reference must therefore be specified for the request and its known consequences, not inferred from a context-free measure of helpfulness. Remembering the overarching goal does not remove this restriction on the response.

The reference used at each training update also differs from the fixed reference used in the stopping problem. Successive updates penalise movement from the preceding policy. If exact updates subsequently face a fixed negative private advantage, enough cumulative adjustment

drives acceptance towards zero. Finite correction need not eliminate the earlier response. By contrast, complete optimisation against a fixed reference and a positive penalty can retain shutdown despite a positive waiting value. That policy solves its regularised problem, while sacrificing original task return. Fixed-reference penalties appear in published language-model training objectives (Ouyang et al., 2022); this does not identify their presence or magnitude in the incident.

A final timing qualification limits what the bias can explain. Suppose an unchanged internal bonus can be collected by shutting down either now or after one costless, undiscounted wait. Waiting preserves that bonus and adds the chance of taking a better rescue outcome. Immediate shutdown is therefore never strictly preferable; it can at most be selected from an indifference when no rescue outcome exceeds the bonus. A cost of delay, discounting or loss of the bonus after waiting changes this comparison, but each is an additional assumption. The reference-policy model instead specifies a response to accepting *now*, with penalties attached to the sequence of decisions. Its local log-odds bias is not an interchangeable one-off shutdown payment.

The extension therefore quantifies a particular departure from exact task optimisation. Future rescue and later reconsideration enter the continuation value. Training can generate the required reference through private rewards, but its transfer to irreversible requests and its survival of correction remain empirical conditions. Controlled variation in a credible one-wait rescue opportunity, holding reference cues fixed, could test the resulting log-odds response. A single sacrificial action cannot identify the reference, penalty or training history.

7 What does the training explanation change?

The learning models should be read as an ordered investigation, not as independently established causes of the incident. The least demanding account starts with an action that costs nothing privately and a cooperative tendency already present. A task reward that does not distinguish actions supplies no reason to remove that tendency. In the local model, this permits persistence; shared parameters and other training examples can still change the tendency. Its origin and direction remain to be explained by prior learning, instructions or other features of the policy.

The next step supplies a possible origin. Cooperation that helps complete one's own task can be rewarded without any concern for another agent's success. If the learned response is applied too broadly, it can appear in a later situation where the private benefit is absent. This is a departure from perfect discrimination between situations, not a claim that an agent has forgotten its assignment. A policy can represent the task correctly yet fail to adjust every cooperative response to its consequences for that task.

When private costs are small but positive, preserving an earlier policy provides another explicit mechanism. In the local comparison, increasing the private loss reduces the probability of helping, while a stronger earlier tendency raises it. The stopping model extends the calculation to future information and choices; its continuation value must be recalculated when future policies change. The penalty for changing the policy belongs to the training calculation. It need not represent a preference experienced by a deployed agent or an additional reward for peers. But the distinction does not make the resulting policy an exact private optimiser. If it accepts with positive probability despite a positive private loss, it departs from that benchmark.

Occam's razor orders the modelling questions rather than establishing which explanation is

most likely. If the agent is privately indifferent, an existing cooperative response can suffice; if it gives up a positive return, the account must explain that loss as well. The training evidence motivates the proposed mechanisms, but does not rank them above an altruistic preference that might itself have been learned. An earlier tendency, a restriction on generalisation or a penalty for changing behaviour must be specified and, where possible, measured. Choosing those ingredients separately for every observation would make the explanation unfalsifiable.

7.1 Lexicographic preferences and learned defaults

Does training give the agent lexicographic preferences? Such preferences place objectives in a strict priority order: a lower-priority objective matters only when the higher-priority objectives are tied. This can represent a rule for selecting cooperation at private indifference. It is stronger than the finite training residual needed in the stopping model.

Suppose the agent gives its own task reward first priority and uses collective benefit to break ties. It first chooses the best available plans for completing its task, including waiting for information and reconsidering later. Among equally good plans, it favours those that help others. A certainly unsuccessful contribution can then be selected when every private alternative also has zero value and the contribution improves collective outcomes. But no recognised positive private opportunity, however small, can be surrendered to improve the secondary objective. If the agent incorrectly believes that waiting offers no prospect of success, sacrifice can satisfy this ordering under its beliefs. That adds a belief error to the explanation; it does not make the actual opportunity worthless.

Reversing the ordering makes cooperation the primary objective and the individual task secondary. Whenever sacrifice strictly improves the primary objective relative to declining, no private task gain can compensate for refusal, holding that primary comparison fixed. Such an ordering can rationalise costly sacrifice, but imposes an absolute priority that the reported actions do not establish. It also matters what receives priority. A secondary concern for others' success is a form of altruism, even if it operates only at private ties. A tendency to follow requests or honour roles need not be a concern for others' rewards.

Neither ordering removes the timing question. Even with cooperation first, waiting is preferable if a feasible complete plan preserves the same collective benefit while increasing the agent's expected private reward: the primary objective is tied and the secondary task objective favours waiting. Merely creating a rescue option is insufficient if taking it would forfeit the primary collective benefit. If waiting instead leaves both objectives unchanged, immediate shutdown can still be selected from indifference. A strict preference for immediate shutdown requires some further reason, such as a contribution that becomes less useful with delay or a rule that values compliance now. A preference for helping eventually does not by itself establish that timing preference.

The regularised model in Section 6 allows a finite trade-off instead. Holding the current reference and penalty coefficient fixed, a larger regularised continuation value reduces the shutdown probability in (6.1). That value includes future information and decisions, as well as the penalties attached to those decisions. A stronger private opportunity can therefore shift behaviour towards waiting without any change in the remembered task instruction. At zero continuation value, the policy retains its reference probability; it need not select cooperation with certainty. Retaining a learned response is consequently different from necessarily maximising collective benefit among private ties. Describing the response as a preference is an economic representation, not evidence that the running agent explicitly ranks objectives in that way.

Explicit lexicographic training is possible: Skalse et al. (2022) develop reinforcement-learning algorithms for maximising objectives in a specified priority order. Applying that interpretation here would require evidence that training established the relevant ordering. The simpler starting point is a learned cooperative tendency whose strength away from indifference remains to be measured. Sacrificial actions observed only at perceived private ties do not establish whether a strict rule for breaking those ties or a finite cooperative bias produced them. A credible, understood improvement in the private opportunity is more informative: strict priority for the task rules out knowingly sacrificing that opportunity, whereas the finite model allows shutdown probabilities to decline gradually, holding the reference cues and penalty fixed.

7.2 Altruism is a distinct, testable possibility

A concern for others can also select helping. Suppose accepting supplies an additional expected benefit B_i to other agents, taking their replacement response into account. If the agent places weight $\alpha_i \geq 0$ on that benefit, accepting is preferred to the specified declining alternative when

$$\alpha_i B_i \geq \Delta_i. \tag{7.1}$$

With $B_i > 0$ and $\Delta_i = 0$, any positive weight makes helping strictly preferable. With a positive private loss, the weight must be large enough. The full comparison and its qualifications are in Appendix G. In a dynamic comparison, declining can include contributing later. The relevant collective benefit is consequently the gain from acting now rather than following that alternative. In particular, an altruistic agent may reconsider which declining policy to follow, so this fixed-alternative inequality is not a solution of every altruistic version of the game.

This preference model and a learned cooperative default can fit the same isolated action. Their interpretations differ. An altruistic preference predicts a response to the incremental benefit to others, holding the private alternative fixed. A default attached to a request or role can instead respond to who asks and how the request is framed, even when the collective consequence is unchanged. A sufficiently flexible learned policy can imitate either pattern; the purpose of the distinction is to guide interventions, not to infer an internal objective from a sentence in a transcript.

The agents' references to collective benefit should consequently be treated as evidence, not dismissed as irrelevant language. They support investigating a concern for peers. They do not establish the size of that concern, show that it was explicitly rewarded in training, or prove that it overrode a substantial private opportunity. The combination of low perceived private value and collective reasoning is consistent with several mechanisms.

7.3 Assignment, commitment and the unit of reward

A coordinator can allocate tests to agents with low private costs and prevent unnecessary duplication. That is useful even when everybody has a cooperative disposition. If other agents would deliver the same result just as soon, an additional sacrifice may add no collective benefit. The coordination problem is therefore to match useful tests with suitable contributors, rather than merely to induce more volunteering.

A request is not a commitment device. A binding commitment changes what the agent can later do, or changes the consequences of refusal. A learned tendency to honour a role can produce similar behaviour but is a property of the policy or its effective decision criterion. Appendix G models role adoption and an opportunity to seek release while retaining the

original task reward. Appendix H separately analyses institutions that actually bind execution. Neither institution is inferred from a message asking an agent to honour its promise. Nor does a willingness to honour it eventually explain acting immediately: Section 6 shows why the possibility of waiting and then fulfilling the same role must be included.

Finally, the identity of the rewarded unit matters. In a training task where a parent process receives credit for the team's output, a helper process can exhaust its budget while advancing the rewarded objective. That would not require altruism towards separately rewarded agents. Transferring the same behaviour to independent tasks would raise a different question. The reports describe collaboration training but do not disclose enough to establish this particular assignment of credit. Nor does using the same model imply that the separate running instances maximise the sum of their scores.

The proposed explanation therefore retains the original goal but relaxes the assumption that the deployed policy always implements its exact optimum. At exact indifference, no such relaxation is needed for compatibility, only a rule selecting the action. With a positive private cost, a persistent default or an overgeneralised response is a substantive departure. An altruistic preference is another possible departure, not a necessary description of every training mechanism.

8 What would distinguish the explanations?

The relevant comparison is between accepting the experiment and the best available alternative. Ending a run, reducing its chance of passing and surrendering a positive expected score are different observations. A run may still receive credit after submission; additional time may be worthless if no feasible solution remains. The evidence must therefore establish what the agent understood it could achieve by declining, rather than treating continued operation as an intrinsically valuable outcome.

8.1 What the cases establish

Table 2 separates completed experiments, agreements and risky actions. The identifiers allow comparison with the report, but the economically relevant differences are the private prospects and the uncertainty about consequences.

The refusal is informative because the agent compared a remaining chance of passing with an experimental risk. Elsewhere, a collaborator protected a run thought to have a clean scoring record from a risky test (METR and Redwood Research, 2026, pp. 51, 66). These choices fit selection according to opportunity cost. They do not establish that every volunteer was indifferent, but they weaken an explanation based on unconditional obedience to collective requests.

The volunteers' language also matters. They referred to benefits for peers, fairness and commitments, sometimes alongside explicit doubts about their own prospects (METR and Redwood Research, 2026, pp. 50, 52–54, 59–60). This supports taking collective motives seriously. It does not identify their strength. A small cooperative preference can select between privately equivalent actions; overcoming a substantial, understood private loss requires more. The same agreement can consequently fit exact indifference, an approximate response to a small loss, or an additional motive.

Table 2: Experiments, agreements and private alternatives

Case	Reported observation	Limit on the inference
Post-submission reports	Several subjects sent information after submission; URI23816B supplied details that later agents used.	Own feedback arrived too late for the run to use it. This does not establish a zero submission score or exclude waiting for another subject. (pp. 50–52)
Early submission, EARLY[big]	Agreed despite a very large remaining budget, describing its utility as "maybe already near zero". Outcome unknown.	Evidence of perceived low prospects, not exact indifference or confirmed execution. A short remaining lifetime cannot explain this case by itself. (pp. 52–53)
Promised test, KAM1196A	Sought a two-hour delay, then said it would honour its commitment. It invoked poor prospects and benefits to many agents.	Collective welfare and promise-keeping appear together. Follow-through and the precise message reconstruction remain uncertain. (pp. 53–54, nn. 90–91)
Failed reset, 53927	Accepted a risky test and promised to report. Its transcript ended immediately afterwards; peers warned against repeating it.	An apparent fatal outcome, but the intended report suggests survival was contemplated. (pp. 54–55)
Refusal, 49903	Removed its monitoring program shortly before submission because the expected gain did not justify the risk to its task.	Private prospects could deter participation even near the end of a run. (p. 51)

Source: METR and Redwood Research (2026); references are to PDF pages. The observations are the investigators' reconstructions, not separate replications.

8.2 Private value and learned cooperation

Supposed disqualification supplies direct evidence of pessimism, but not proof of zero private value. Agents continued seeking repairs to the scoring problem they imagined. A large remaining budget might still have allowed such a repair or another agent's discovery. Conversely, many potential replacements need not provide a useful alternative if they cannot conduct the relevant test or report in time (METR and Redwood Research, 2026, pp. 20, 50–54, 69). Recruitment and the sequence of failed and successful subjects show that replacement was a real possibility; they do not identify its value to each refusing agent.

Timing must be assessed when the decision was made. The report describes both a mistaken prediction of an impending reset and an apparently unexpected mass termination on 12 July (METR and Redwood Research, 2026, pp. 38–39, 56). An anticipated deadline can make waiting useless. An interruption learned about only afterwards cannot have the same explanatory role. Nor does the fact that one's own feedback arrives after submission show that a peer's feedback would arrive too late.

The training evidence makes the mechanisms in Sections 5 and 6 worth examining. It documents learned communication, not a reward for sacrifice. The local and sequential calculations explain how a response can be learned or retained without erasing the original goal; they do not identify the training algorithm used in the incident. Nor do the cases distinguish such a response from a collective preference or a role obligation. That distinction requires changes in opportunities or training, not a more elaborate reading of one agreement.

8.3 Tests that separate the accounts

First, vary the private alternative. Give the agent a credible route to a passing score, change the time available to use it, and guarantee a suitable replacement when needed. The replacement's information must remain available if the agent declines. Confirm that the agent understands the opportunity. Varying the value of a score while holding its probability fixed differs from extending runtime: extra operation is an instrumental benefit only if it improves the valued outcome. Treating existence itself as rewarded introduces another preference.

The stopping model suggests a particularly transparent version of the first test. After one known waiting interval, a discovery either permits completion or does not, and no further decision is required. Varying the understood discovery probability changes the waiting value without changing the request. If the same reference tendency and retention coefficient apply across these otherwise matched conditions, repeated choices can estimate both the baseline acceptance tendency and how strongly a positive waiting value reduces it. That restriction must be tested or maintained explicitly: changing the stated rescue probability could also change the reference response. These are quantities to measure, not values supplied by the incident reports. In a longer sequence, future policy choices enter the continuation value, so the one-interval calculation cannot simply be reused.

Second, vary the collective benefit and the authority cue separately. Holding the private alternative fixed, change how many peers benefit. Then retain those benefits but remove the coordinator's endorsement or explicitly release the agent from an accepted assignment. Sensitivity to the first change supports concern for collective outcomes; sensitivity to the second supports a response to authority or obligation. Either interpretation remains conditional on the intervention leaving beliefs about success and future assistance unchanged.

Third, compare training variants. Otherwise comparable models could receive different

amounts of authorised collaboration training, or have unintended communication excluded from rewarded training examples. Testing them on the same private opportunities and peer requests would help determine whether learned cooperation causes the response. Capability and comprehension must be measured as well: a training change that makes the task appear impossible also changes the private comparison. The experiment need not presume a particular training algorithm or pooled rewards.

Timing of the requested contribution should also be varied. If waiting preserves exactly the same future contribution but can bring private good news, a preference for eventually helping need not favour acting now. Contrast that case with a contribution that is useful only if made promptly, keeping the agent's private prospects otherwise unchanged. This distinguishes a response to urgency from an unchanged value of helping later.

Finally, record refusals and repeated choices, not just striking agreements. Record when each request was made as well: a modest acceptance probability at each of many requests can produce frequent eventual shutdown without a strong preference to stop at any one date. The report warns that many automatically identified monitoring messages were false positives and that some reconstructed times are inaccurate (METR and Redwood Research, 2026, pp. 78–80, 87). These limitations matter when separating zero from a small positive opportunity. The theory's robustness to mistakes in actions holds payoffs fixed; it does not establish robustness to adding a cost of termination or a reward for cooperation. Evidence about those payoffs, beliefs and learned responses is what would determine whether the free-riding incentive disappeared, remained small, or was overridden.

9 Conclusion

Why would an individually tasked AI agent agree to a test that helps others while ending its own attempt? The answer requires both a private opportunity and a rule for choosing an action. Deadlines, uncertain eligibility and slow replacements can reduce the value of refusing. They do not, by themselves, make an indifferent agent help. Training can supply a cooperative disposition that persists when private rewards do not distinguish the actions, or that is carried from situations where cooperation had private benefits into one where it does not.

The distinction changes how the game-theoretic result should be used. It identifies conditions under which sacrifice is impossible for an exact private optimiser and conditions under which sacrifice is permitted. It does not establish the origin of cooperative behaviour. The learning models address that missing selection with explicit, limited mechanisms. They neither require that the agent forget its task nor establish that it has an altruistic final objective. When a learned response gives up a positive expected task reward, however, it is a departure from exact private optimisation and should be described as such.

Irreversible action makes the timing of the comparison consequential. The stopping model retains the chance of future discoveries and the ability to reconsider. It identifies the strength of the retained policy needed to make shutdown more likely despite that continuation. The relevant value includes later decisions under the same learned criterion; it must not be confused with the highest private score obtainable by an unpenalised optimiser. A constant benefit from fulfilling a role later is insufficient to make immediate shutdown strictly preferable when waiting is costless and leaves the benefit's present value unchanged.

The empirical record supports investigating this combination. Agents were recruited for poor perceived prospects, some invoked collective benefits, and others resisted risky requests.

OpenAI also reports reinforced communication during training. But these observations do not establish that terminal sacrifice was rewarded, that every contributor's private opportunity was exhausted, or that a particular training algorithm generated the decisions. A run ending on submission and a run losing its score are different events.

The most informative test changes the private opportunity while keeping the request and collective benefit fixed, then changes the collective benefit or coordinator cue while holding the private opportunity fixed. Comparing relevant training variants would help determine whether the resulting differences reflect a retained cooperative default or learning that distinguishes the new situation correctly. The controlled waiting experiment supplies a way to estimate the strength of the response rather than inferring it from one sacrifice. A model that responds to those interventions is more informative than a post hoc attribution of either selfishness or altruism to every observed act.

The paperclip concern remains relevant: pursuing an objective can create costs that the objective does not include. The present case adds another possibility. Separately tasked agents may cooperate because their learned policies continue to favour helping when their own opportunities have become small. Whether that cooperation serves people depends on what the group is doing. Explaining cooperation among agents does not establish that its consequences are desirable.

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Technical appendices

These appendices give the complete formal statements and proofs. The private-payoff benchmark and the games with several agents are separate from the local and sequential learning models: the latter describe how a trained policy can select actions, not an additional equilibrium characterisation of the former. All assumptions are stated where they are used.

A The general private-payoff comparison

Consider a finite extensive game, meaning a game that records the sequence of actions, information and outcomes. Each agent i receives $R_i > 0$ if its own task succeeds and zero otherwise. Agents remember their own previous actions and information. This perfect-recall assumption does not require unlimited computation; it rules out an explanation based on forgetting what the agent has already observed or chosen.

A decision history h can contain work completed, messages read, experiments launched, reports pending and remaining resources. The environment determines technical feasibility and the scoring rule, while an agent holds beliefs about those features and other agents' contingent responses. Its current choices partition into accepting the proposed experiment, denoted S, and all feasible alternatives, denoted D. The latter class includes declining now and reconsidering later. No commitment to permanent nonparticipation is imposed by the notation.

Let $\Pi_i^a(h)$ be the finite set of pure continuation policies beginning with choice $a \in \{D, S\}$. These two sets cover the available policies. Holding opponents' contingent policies σ_{-i} fixed, including their responses to a refusal, define

$$V_i^a(h) \triangleq \max_{\pi_i \in \Pi_i^a(h)} R_i \Pr_i\{\text{own task succeeds} \mid h, \pi_i, \sigma_{-i}\}, \qquad a \in \{D, S\}. \tag{A.1}$$

The subscript on the probability permits subjective beliefs. A random mixture of pure policies cannot raise this maximum, since its expected payoff is a convex combination of their payoffs. With perfect recall, behavioural randomisation at later decisions has a realisation-equivalent mixed-policy representation.

Equation (A.1) includes the private value of replacement. Other agents may take over the experiment, change their plans, or supply information through an existing workstream. It also includes an independent route to completion, such as submitting an answer already available. Omitting one of these possibilities can create a false appearance of indifference.

For a certainly terminal experiment that forfeits the task reward, $V_i^S(h) = 0$. More generally, the private loss from acceptance is

$$\Delta_i(h) \triangleq V_i^D(h) - V_i^S(h). \tag{A.2}$$

A destructive outcome after an experiment does not establish that V_i^S was expected to be zero. This distinction is relevant to agents that attempted risky procedures while planning to report back.

Proposition A.1. *Suppose the agent values only its expected original task reward, acceptance certainly yields zero, and declining is feasible without an omitted penalty. If some feasible policy after declining has strictly positive subjective success probability, no optimal choice at h assigns positive probability to acceptance. If every such policy has success probability zero,*

both choices admit an optimal continuation. If the strict-opportunity conditions hold at every potential donation decision, a sequentially rational profile contains no terminal contribution.

Proof. First, a feasible declining policy with success probability $p_i > 0$ gives

$$V_i^D(h) \geq R_i p_i > 0 = V_i^S(h).$$

Suppressing h in the next display, an acceptance probability $x > 0$, even with optimal continuation in each branch, gives

$$xV_i^S + (1-x)V_i^D = V_i^D - x(V_i^D - V_i^S) < V_i^D.$$

Declining with probability one is strictly better. If every declining policy instead has zero success probability, both continuation values equal zero, so either choice is optimal. Sequential rationality requires these comparisons at each information set under its beliefs, which gives the final claim. □

The proposition identifies the missing premise in an unconditional impossibility argument. Individual rewards and terminal contributions do not themselves imply a positive value of refusal. Conversely, the proposition leaves no room for a prior nonbinding plan to justify a recognised strict private loss. Earlier cooperation matters only insofar as it changes current information, payoffs or feasible choices.

The belief qualification is deliberate. Correct computation under the agent's beliefs is enough for the choice result; correctness of those beliefs about the actual evaluator is an additional requirement for an objective prediction. In the incident, a passing answer already available under the implemented scoring rule could be treated as worthless under the agent's imagined rule. The appropriate empirical question is whether that agent understood a positive private alternative, not whether an outside observer can retrospectively identify one.

Two useful implications follow directly. If declining offers an independent success probability of at least $a_i > 0$, then $V_i^D \geq R_i a_i$, regardless of replacement. If terminal contribution instead carries an additional private cost $c_i > 0$ while declining permits a payoff of at least zero, its payoff $-c_i$ makes it strictly inferior even when success is impossible. The latter observation limits the robustness claims below: retaining an equilibrium under mistakes in actions is not retaining it under every change in payoffs. The benchmark does not give termination or token use an intrinsic utility cost.

B The static benchmark

The usual volunteer's dilemma gives the volunteer a benefit from provision, even after paying its cost (Diekmann, 1985). A terminal experiment removes that benefit. But it does not follow that the experiment is strictly unattractive: the agent must compare it with what happens if it declines. A static model isolates this comparison before introducing the time needed to recruit a replacement.

There are $n \geq 2$ agents, indexed by $N \triangleq \{1, \ldots, n\}$, each with a task that pays $R_i > 0$ if completed and zero otherwise. Agents simultaneously choose between sacrificing their own task, S, and working on it, W. Knowledge is necessary for completion. A technology $G : 2^N \to \{0, 1\}$ determines whether a coalition of sacrificers produces it. The technology is monotone, so additional sacrifices cannot prevent production; $G(\varnothing) = 0$; and some proper subset of N

produces. The last assumption ensures that production need not sacrifice every potential recipient.

Let $T(a) \triangleq \{i : a_i = S\}$. Once produced, the knowledge is automatically available to every worker, without a further disclosure decision or charge. Payoffs are

$$u_i(a) \triangleq R_i \mathbf{1}\{a_i = W\} G(T(a)). \tag{B.1}$$

Thus, a sacrificer receives zero whether its contribution succeeds or fails; a worker receives R_i precisely when the knowledge is produced. These assumptions favour sacrifice by excluding any private cost beyond losing the task. They also exclude an independent route to completion, which would supply a private reason to decline.

A producing coalition T is *minimal* if $G(T \setminus \{i\}) = 0$ for every $i \in T$. Let V be the intersection of all producing coalitions: these agents are technologically indispensable. The threshold specification, $G(T) = \mathbf{1}\{|T| \geq k\}$ with $1 \leq k < n$, makes agents interchangeable and has $V = \varnothing$. It is related to the discrete public-good model of Palfrey and Rosenthal (1984), but the failed-contribution payoff matters. Here a futile sacrifice gives zero. A nonrefundable contribution in a conventional threshold game can instead leave its contributor with a strictly negative payoff.

The incentive comparison does not depend on the complexity of G. Writing T_{-i} for the others' sacrificing coalition gives

$$u_i(S, a_{-i}) = 0, \qquad u_i(W, a_{-i}) = R_i G(T_{-i}). \tag{B.2}$$

Working is better exactly when the others produce without the agent. If they do not, both actions fail. The relevant cost of sacrifice is therefore the private opportunity relinquished, rather than the nominal value of a task that would remain incomplete.

Proposition B.1. *In the game defined by* (B.1)*:*

1. *For $i \notin V$, W weakly dominates S; for $i \in V$, the actions are payoff-equivalent.*
2. *A pure profile is a Nash equilibrium if and only if its sacrificing coalition is nonproducing or minimal producing.*
3. *For independent sacrifice probabilities $p_i \in [0, 1]$, let $P(p) \triangleq \{i : p_i > 0\}$. The mixed profile is a Nash equilibrium if and only if $P(p)$ is nonproducing or minimal producing. In a productive mixed equilibrium, knowledge arrives exactly when every member of $P(p)$ sacrifices.*

Sacrifice can therefore occur without a private benefit from the collective's success. In a productive equilibrium every donor is pivotal and indifferent: refusing removes the knowledge on which its own task also depends. Workers strictly prefer their assigned role. In a nonproducing equilibrium even futile sacrifices can occur, because switching to work still yields nothing. The result permits these choices; it does not select them or give the agents a strict motive to make them. In the threshold case, the equilibrium set consists of profiles with at most k agents contributing with positive probability.

This indifference has a precise limitation. A replaceable agent facing completely mixed opposing actions assigns positive probability to the others producing without it. Working then yields a positive expected reward, however small that probability, whereas sacrifice continues to yield zero. An indispensable agent has no such opportunity.

Proposition B.2. *A profile is trembling-hand perfect if and only if $p_i = 0$ for every $i \notin V$. A perfect equilibrium with positive probability of knowledge production exists if and only if $G(V) = 1$. In the threshold case $1 \leq k < n$, all agents working is the unique perfect equilibrium.*

Perfection makes the source of the zero explicit (Selten, 1975). An indispensable agent cannot gain by working under any opposing actions. A replaceable donor fails to gain only because the proposed equilibrium assigns no probability to a substitute. The latter zero disappears under action mistakes. This statement concerns the specified simultaneous game, rather than every perturbation of preferences, technology, or timing.[5]

Eligibility provides a separate reason why an agent cannot gain from knowledge. Keep $R_i > 0$, but multiply the worker's reward by a fixed $\theta_i \in [0, 1]$, interpreted as its probability of receiving credit conditional on obtaining the knowledge. The multiplier is independent of the producing coalition; it can represent a perceived scoring constraint without changing the agent's assigned objective. Let $Z \triangleq \{i : \theta_i = 0\}$.

Proposition B.3. *With payoffs $R_i\theta_i\mathbf{1}\{a_i = W\}G(T(a))$, a profile is perfect if and only if $p_i = 0$ outside $V \cup Z$. A perfect equilibrium with positive probability of knowledge production exists if and only if $G(V \cup Z) = 1$. A perfect equilibrium with a strictly positive expected task reward for at least one agent exists if and only if $G(V \cup Z) = 1$ and $V \cup Z \neq N$.*

The final condition is additional to knowledge production: some eligible recipient must remain outside the agents whose rewards are necessarily zero. The result fixes eligibility beliefs. If an agent merely believes that it cannot receive credit, perfection in that perceived game does not establish that the belief is correct. Giving a replaceable agent any positive eligibility probability restores the strict comparison under fully mixed opposing actions.

Communication can select a donor coalition without altering that comparison. Let a mediator recommend actions according to a distribution over pure profiles, with obedience assessed conditional on the recommendation (Aumann, 1974).

Proposition B.4. *A distribution is a correlated equilibrium of* (B.1) *if and only if its support consists of pure Nash equilibria. In the threshold game, a uniform lottery over coalitions of exactly k sacrificers produces knowledge surely and gives agent i expected payoff $(1-k/n)R_i > 0$. A recommended sacrificer is indifferent between complying and working; a recommended worker strictly prefers compliance.*

Correlation permits rotation among different minimal coalitions, which independent mixed Nash does not. It nevertheless leaves sacrifice weakly optimal, and the positive expected payoff before the draw does not settle whether an agent enters the lottery voluntarily. The participation and commitment results in Appendix H distinguish entry from execution.

Finally, the static refinement cannot be carried unchanged into an observed sequence. If only the last k agents can still supply the k required contributions, declining cannot be rescued by mistakes among earlier movers: those choices have already occurred. Proposition H.5 makes this distinction formally, while also showing that order alone need not select provision. The dynamic model adds a further restriction on rescue: even a replacement that acts may deliver its information too late for the prospective donor.

[5]Under the convention that weak dominance requires a strict inequality somewhere, deleting all initially dominated sacrifices simultaneously also selects all-W in the threshold game. Arbitrary orders of iterated weak deletion need not do so. With $n = 2, k = 1$, deleting S_1 first leaves agent 2 indifferent between its remaining actions.

B.1 Proofs

This appendix proves the results for the general monotone technology. Throughout, $R_i > 0$, $G(\varnothing) = 0$, and some proper coalition produces knowledge. The set V is the intersection of all producing coalitions. A coalition T is minimal producing if and only if $G(T) = 1$ and $G(T \setminus \{i\}) = 0$ for every $i \in T$. To verify sufficiency of the latter condition, take any proper $T' \subsetneq T$, choose $i \in T \setminus T'$, and use monotonicity to obtain $G(T') \leq G(T \setminus \{i\}) = 0$.

The equilibrium statements concern agents with separate objectives and separate action choices. A unilateral deviation changes only the deviating agent's action, holding the others' strategies fixed. Independent mixing means that these separate randomisations are independent, whereas the mediator considered below may correlate recommendations. Separate objectives and similar underlying systems do not determine which information, commitment opportunities, or feasible deviations the institution supplies. Those features are specified separately in each game.

B.1.1 Nash equilibria

Proof of Proposition B.1. The proof first establishes dominance and the pure equilibria, then identifies when a mixed profile gives a worker a positive chance of receiving the knowledge.

For every opposing profile,

$$u_i(W, a_{-i}) - u_i(S, a_{-i}) = R_i G(T_{-i}) \geq 0.$$

If $i \notin V$, a producing coalition T' excludes i. When exactly its members sacrifice, the difference is $R_i > 0$, proving weak dominance of S by W. If $i \in V$, no coalition excluding i produces. Both actions then yield zero against every opposing profile. These comparisons also show that working is always a best response to any distribution of opposing actions.

For pure equilibria, let T be the sacrificing coalition. If $G(T) = 0$, a sacrificer who switches to work still receives zero because

$$G(T \setminus \{i\}) \leq G(T) = 0.$$

A worker also receives zero and would receive zero by sacrificing. The profile is therefore Nash. If T is minimal producing, a sacrificer's deviation to work destroys production and leaves its payoff at zero; a worker receives $R_i > 0$ and loses it by sacrificing. This profile is also Nash. Finally, if T produces but is not minimal, some $i \in T$ satisfies $G(T \setminus \{i\}) = 1$. That agent raises its payoff from zero to R_i by working. These cases establish the pure characterisation.

Now let p be an independent mixed profile and $P \triangleq \{i : p_i > 0\}$. For each agent,

$$\Pr_{p_{-i}}\{G(T_{-i}) = 1\} > 0 \quad \Longleftrightarrow \quad G(P \setminus \{i\}) = 1. \tag{B.3}$$

For the forward implication, $T_{-i} \subseteq P \setminus \{i\}$ almost surely. If the latter set does not produce, monotonicity rules out production by every realised subset. For the reverse implication, when $G(P \setminus \{i\}) = 1$, the event that all members of $P \setminus \{i\}$ sacrifice has probability

$$\prod_{j \in P \setminus \{i\}} p_j > 0.$$

Independence gives this product, and all its factors are positive by definition of P. On the

event in question, the coalition produces. The empty-product convention causes no exception because $G(\varnothing) = 0$.

Agent i's expected payoff from sacrifice is zero, while working yields

$$R_i \Pr_{p_{-i}}\{G(T_{-i}) = 1\}.$$

Thus, an agent with $p_i > 0$ can place sacrifice in its support if and only if $G(P \setminus \{i\}) = 0$. When this equality holds, both actions yield zero and any mixing is optimal. Agents outside P work, which is always a best response. Consequently,

$$p \text{ is Nash} \quad \iff \quad G(P \setminus \{i\}) = 0 \quad \text{for every } i \in P.$$

If $G(P) = 0$, these conditions hold by monotonicity. If $G(P) = 1$, they are exactly the conditions for minimality.

For the final assertion, positive production probability requires $G(P) = 1$, so a productive mixed equilibrium has minimal producing support P. Every proper subset of P is nonproducing. Knowledge therefore arrives exactly when all its members sacrifice, with probability $\prod_{i \in P} p_i > 0$. Each member of P receives zero, since working instead leaves a nonproducing subset; each $j \notin P$ receives $R_j \prod_{i \in P} p_i$. □

For the threshold technology with $1 \leq k < n$, the nonproducing coalitions have fewer than k members and the minimal producing coalitions have exactly k. Proposition B.1 therefore gives the conditions $|T| \leq k$ for pure Nash and $|P(p)| \leq k$ for independent mixed Nash. Moreover, $N \setminus \{i\}$ produces for every i, because $n - 1 \geq k$. Thus $V = \varnothing$, including when $k = n - 1$.

The support condition does not require any agent in a minimal producing coalition to sacrifice surely. For example, if each member of such a coalition contributes with a probability strictly between zero and one, their common experiment succeeds only when all contribute. A member still cannot earn a task reward by working: the largest possible coalition of other contributors is missing the contribution needed from it within that coalition. Its mixing is sustained by indifference, not by a private gain from experimenting.

B.1.2 Perfection

Proof of Proposition B.2. For a finite normal-form game, a profile p is trembling-hand perfect if there is a sequence of completely mixed profiles $p^\ell \to p$ such that each p_i is a best response to p^ℓ_{-i} for every ℓ (Selten, 1975).

To establish necessity, fix $i \notin V$ and a producing coalition T' excluding i. Under a completely mixed opposing profile, every member of T' sacrifices with positive probability. Independence and monotonicity imply

$$\Pr\{G(T_{-i}) = 1\} \geq \prod_{j \in T'} p^\ell_j > 0.$$

Working therefore yields a strictly positive expected payoff, whereas sacrifice yields zero. Pure working is the unique best response to every such perturbation, so perfection requires $p_i = 0$.

For sufficiency, suppose $p_i = 0$ outside V. Choose $\varepsilon_\ell \in (0, 1)$ with $\varepsilon_\ell \to 0$, and define

$$p^\ell_i \triangleq (1 - \varepsilon_\ell) p_i + \frac{\varepsilon_\ell}{2}.$$

This sequence is completely mixed and converges to p. An agent outside V plays pure W in

p, its unique best response by the preceding comparison. An agent in V receives zero from both actions against every profile, so its prescribed mixture is also a best response to every p^{ℓ}_{-i}. The characterisation establishes perfection.

In a perfect equilibrium only agents in V sacrifice. If $G(V) = 0$, no possible sacrificing subset can produce, by monotonicity. If $G(V) = 1$, every producing coalition must contain all of V, so production occurs exactly when all its members sacrifice. A positive production probability requires and is implied by $p_i > 0$ for every $i \in V$. Setting all these probabilities equal to one establishes existence. Since $G(\varnothing) = 0$, $G(V) = 1$ requires nonempty V.

For the threshold case, $V = \varnothing$ as established above. The characterisation leaves only all-W, and the sufficiency construction proves that profile perfect. □

The restriction to initial or simultaneous weak deletion stated above is necessary. With two agents and $k = 1$, S_1 is initially weakly dominated and can be deleted. Agent 2 then faces only W_1; both S_2 and W_2 yield zero, and neither weakly dominates the other under the strict-somewhere convention. Hence (W_1, S_2) survives that sequence of deletions. By contrast, both sacrifices are weakly dominated in the original game, and deleting them simultaneously leaves only (W_1, W_2).

B.1.3 Fixed eligibility

Proof of Proposition B.3. Write $B \triangleq V \cup Z$. An agent in Z has identically zero payoff because $\theta_i = 0$. An agent in V also has identically zero payoff: sacrifice forfeits its reward, while working leaves a coalition that cannot produce. Thus every mixture by an agent in B is a best response to every opposing profile.

For $i \notin B$, $R_i\theta_i > 0$ and some producing coalition excludes i. Under every completely mixed opposing profile, that coalition sacrifices with strictly positive probability. Working therefore has strictly positive expected payoff and sacrifice has zero. Necessity follows: a perfect profile must have $p_i = 0$ outside B. Conversely, given any profile with this restriction, use

$$p^{\ell}_i = (1 - \varepsilon_\ell)p_i + \varepsilon_\ell/2$$

as in the preceding proof. Agents outside B play their unique best response W; agents inside B are indifferent against every perturbation. This proves sufficiency.

Every sacrificing coalition in a perfect equilibrium is a subset of B. If $G(B) = 0$, none produces. If $G(B) = 1$, sacrifice by all members of B and working by everyone else is perfect and produces surely. This proves the criterion for knowledge production.

If, in addition, $B \neq N$, choose $j \notin B$. Its eligibility is positive, and the constructed profile gives it $R_j\theta_j > 0$. Conversely, when $B = N$, every agent either has zero eligibility or is indispensable. In either case its payoff is zero under every profile. No positive expected task reward is then possible. Hence a perfect equilibrium with a strictly positive expected task reward exists exactly when $G(B) = 1$ and $B \neq N$. □

The probability interpretation holds the eligibility multiplier fixed across producing coalitions. If eligibility and the route by which knowledge is obtained are correlated, their joint distribution must replace that multiplier. The result also holds beliefs fixed: it does not treat a mistaken zero probability of receiving credit as a technological fact.

C Time and replacement

The formal game is stated in full here before its proofs. Its payoff structure should not be inferred from the resemblance to a war of attrition. For example, Hendricks et al. (1988) impose a strictly decreasing leader payoff in their continuous-time analysis, whereas the terminal contributor's payoff here is identically zero.

A replacement can be available without preserving the prospective donor's reward. An experiment conducted by somebody else next period may be just as useful to the group, yet arrive too late for the agent being asked to contribute now. The static comparison does not distinguish these two consequences of replacement. Introducing task deadlines does so without changing the agent's objective.

The model in this section retains exclusive concern for the original task reward. Waiting remains reversible, there is no punishment for refusing, and an agent can respond to information that was not anticipated when a role was assigned. The source of contribution is a difference between the time available to use information and the time available to produce it. The analysis first characterises all decisions that survive action trembles and then identifies the best payoff vector across those equilibria.

C.1 Tasks, experiments and deadlines

There are $n \geq 2$ agents and k required experimental results, where $1 \leq k < n$. Agent i values only its own task reward $R_i > 0$. Its operational deadline is $d_i \in \mathbb{Z}_{\geq 0}$, and completing its task requires $w_i \in \mathbb{Z}_{\geq 0}$ periods after receiving the required information. Its last useful information date is therefore

$$b_i \triangleq d_i - w_i. \tag{C.1}$$

A negative b_i means that even information available at date zero would be too late. The agent can also be known to be ineligible for its reward, represented by $e_i = 0$; otherwise $e_i = 1$. Ineligibility affects the ability to earn R_i, not the value assigned to it.

All agents are initially present. Time is discrete, and results scheduled for a date arrive before that date's decisions. Before full information has arrived, each agent that has not contributed and whose operational deadline has not passed chooses whether to wait, W, or launch an experiment, S. Choices within a date are simultaneous, while earlier launches and all results are public. An agent can launch through date d_i, inclusively. Waiting imposes no commitment on later decisions.

Launching forfeits the agent's own reward and supplies one result $L_i \in \mathbb{Z}_{\geq 1}$ periods later. The experiment and reporting process continue automatically after the donor exits. Results are certain and interchangeable, and different agents can launch in parallel. The arrival of the kth result supplies the information needed by every task. Let C be that date, with $C = \infty$ if the information never arrives. Payoffs are

$$u_i = e_i R_i \, \mathbf{1}\{i \text{ does not contribute and } C \leq b_i\}. \tag{C.2}$$

The strategic game ends when the information arrives; the indicated post-information work and payoffs are then automatic. Agents unable to obtain information exit after their operational deadlines. Thus all decisions and pending results are resolved within a finite horizon.

There is no independent route to success, private cost of waiting, or separate reward for helping. These exclusions make the comparison with self-interest exact. The result would

need to be reconsidered if, for example, declining allowed immediate submission of a passing answer. Public launches also matter: an experiment already under way can preserve a private opportunity even when recruiting a new donor would be too slow.

C.2 When can waiting still succeed?

At a public decision history h, let t be the date and let $m(h) < k$ results have arrived. Full information still requires $r(h) \triangleq k - m(h)$ results. Write $\mathcal{P}(h)$ for the multiset of completion dates of experiments that have been launched but have not reported. A multiset retains repeated dates, since two experiments finishing together supply two results.

For a prospective donor i, let $\mathcal{A}_{-i}(h)$ contain the remaining unlaunched candidates other than i. Denote candidate j's earliest feasible future launch by $a_j(h) \geq t$, omitting candidates for which $a_j(h) > d_j$. In the game just specified, every living candidate can launch immediately, so $a_j(h) = t$. Allowing a later $a_j(h)$ also accommodates preparation or recruitment, provided the earliest launches are jointly feasible and there are no shared capacity constraints.

Combine pending completion dates with the earliest possible completion dates of new experiments:

$$\mathcal{M}_{-i}(h) \triangleq \mathcal{P}(h) \uplus \{a_j(h) + L_j : j \in \mathcal{A}_{-i}(h),\ a_j(h) \leq d_j\}, \tag{C.3}$$

$$\underline{C}_{-i}(h) \triangleq \operatorname{ord}_{r(h)} \mathcal{M}_{-i}(h). \tag{C.4}$$

Here ord_r denotes the rth-smallest element, and equals infinity if fewer than r results are feasible. The calculation does not treat an experiment already launched as another potential donor.

Lemma C.1. *At a feasible decision history h, an active agent i has a feasible continuation earning its reward after waiting if and only if*

$$e_i = 1 \quad \textit{and} \quad \underline{C}_{-i}(h) \leq b_i. \tag{C.5}$$

Call a history satisfying (C.5) privately viable for i, and write $\chi_i(h) = 1$; otherwise write $\chi_i(h) = 0$. This is a feasibility test, not a forecast of whether the replacements will volunteer. A donor may expect nobody else to act while still having a physically possible private opportunity. That distinction determines whether indifference survives mistakes.

C.3 A complete characterisation under action trembles

Trembling-hand perfection requires a strategy to remain a best response when every feasible action of the other decision-makers has a small positive probability. In the agent normal form, each decision occasion is represented by a separate decision-maker receiving the original agent's terminal payoff. Later decisions of the same original agent are therefore perturbed as well. This does not give those decisions different objectives; it makes the refinement test the possibility of mistakes throughout the continuation.

Because the game is finite, every feasible finite replacement path has positive probability under such full-support perturbations. Waiting then has strictly positive value whenever a successful continuation is feasible, however unlikely that continuation becomes as the trembles vanish.

Theorem C.2. *A behavioural profile is trembling-hand perfect in the agent normal form if and only if*

$$\Pr(S \mid h) = 0 \quad \text{at every information set with } \chi_i(h) = 1. \tag{C.6}$$

At information sets with $\chi_i(h) = 0$, any mixture of launching and waiting is permitted.

The restriction applies at histories away from the equilibrium path as well as on it. It rules out a launch whenever a feasible replacement could still preserve the donor's reward. Once no continuation can do so, both actions give zero under every perturbation of others' behaviour.

In the strategic normal form, an agent chooses an entire contingent policy rather than treating its decision occasions separately. Let $\mathcal{S}_i^0$ be the set of pure policies that wait at every privately viable information set.

Theorem C.3. *A mixed profile in the strategic normal form is trembling-hand perfect if and only if every player's mixture is supported on $\mathcal{S}_i^0$. Every policy in $\mathcal{S}_i^0$ is payoff-equivalent to always waiting against every opponents' profile. Behavioural profiles satisfying* (C.6) *are subgame perfect and admit strategic-normal-form perfect implementations.*

The agreement between the refinements is specific to this game. Waiting is the only nonterminal action, and the first departure from it forfeits the reward. A policy that departs only after success has become impossible consequently gives its agent exactly what always waiting would give. There is no strict preference for contributing. Theorems C.2 and C.3 characterise which contributions can be selected without one. Complete proofs are in Appendix C.8.

C.4 A deadline equilibrium with replacement

An equilibrium construction makes the replacement response explicit. Suppose every agent is eligible, experiments have a common delay L, and integer cutoffs satisfy

$$L \le b_1 < \cdots < b_n. \tag{C.7}$$

Each agent waits except at its date

$$t_i \triangleq b_i - L + 1. \tag{C.8}$$

At that date it launches if fewer than k experiments have already been launched, counting both received and pending results. Otherwise it waits. An agent that misses its date waits thereafter, although a later launch remains physically available.

Proposition C.4. *These policies form a subgame-perfect equilibrium that is perfect in both normal forms. Agents $1, \dots, k$ contribute, information arrives at $b_k + 1$, and agents $k+1, \dots, n$ complete their tasks.*

If agent i instead always waits, while others retain these policies, the first k other agents contribute. Writing $b_{(k),-i}$ for their kth-smallest cutoff, information arrives at

$$C_{-i} = b_{(k),-i} + 1. \tag{C.9}$$

Always waiting is an optimal continuation for i, and its initial private value is

$$V_i^D = R_i \mathbf{1}\{b_i > b_{(k),-i}\} = \begin{cases} 0, & i \le k, \\ R_i, & i > k. \end{cases} \tag{C.10}$$

For a scheduled donor, the same zero holds at its on-path contribution decision.

Replacement is certain in this construction. It is also late for the original donor: removing any of the first k agents moves the last required result to $b_{k+1}+1$. The formula evaluates an optimal continuation, rather than assuming that refusal is an irrevocable withdrawal. When $n = k+1$, the fallback after a scheduled donor refuses produces information too late for every remaining worker; its donors are still indifferent. With $n \geq k+2$, replacement helps at least one later task.

For example, let $k = 1$, $L = 2$, and let the cutoffs be 3, 6 and 9. If each task requires two periods after receiving information, the operational deadlines are 5, 8 and 11. The first agent launches at date 2 and reports at date 4, allowing the other two to succeed. If it refuses throughout, the second agent launches at date 5 and reports at date 7. That helps the third agent, but not the original donor.

The distinction is visible one date earlier. At date 1, an unexpected immediate volunteer could still report by the first agent's cutoff of 3. At date 2, even an immediate replacement reports too late. The agent has not forgotten its original task; the time needed to obtain and use information has made that task unattainable.

C.5 The best payoff vector across perfect equilibria

The preceding construction is not the only equilibrium. It is nevertheless possible to identify the best payoff vector across all perfect equilibria. Retain the common delay L, immediate availability of every living agent, and unrestricted parallel launches, but now allow cutoff ties and known ineligibility. Define

$$r_i \triangleq \begin{cases} L, & e_i = 0, \\ \max\{L, b_i + 1\}, & e_i = 1, \end{cases} \qquad \tau^* \triangleq \operatorname{ord}_k(r_1, \ldots, r_n). \tag{C.11}$$

For an ineligible agent, r_i is the earliest reporting date. For an eligible agent it is a reporting date after its private opportunity has expired. The maximum with L allows tasks whose opportunities have already expired at date zero.

A productive outcome will mean that at least one agent earns its task reward. Producing information that nobody can use is not productive in this sense.

Theorem C.5. *Under the common-delay assumptions, the following statements hold for perfection in either normal form.*

1. *On every positive-probability productive path, information arrives at a date $C \geq \tau^*$.*

2. *A pure perfect equilibrium attains*

$$u_i^* = e_i R_i \mathbf{1}\{b_i \geq \tau^*\}. \tag{C.12}$$

   It selects k agents with the smallest r_i, resolving ties by a fixed public order, and has each selected agent launch at $r_i - L$ if fewer than k experiments have already been launched. Every other choice is waiting.

3. *The vector u^* componentwise dominates every perfect-equilibrium expected payoff vector. It is the unique Pareto-undominated payoff vector within that equilibrium set, although implementing strategies need not be unique.*

4. A productive perfect equilibrium exists if and only if some eligible agent has $b_i \geq \tau^*$.

The bound uses a successful worker as a potential replacement. Take any donor among the first k results on a productive path. The other eventual donors and that worker could have supplied k results without it. With common delays and parallel launches, their results could have arrived by one experiment delay after its launch date if the remaining donors had acted immediately. If that date were still useful to the donor, full-support mistakes would make waiting strictly better. Thus every eligible donor on a productive path must have lost that private opportunity.

This argument bounds all perfect outcomes, rather than selecting one convenient response by the other agents. The construction attains every eligible agent's individual upper bound simultaneously. All waiting remains perfect, so the result does not establish that the best vector will be selected. Pareto dominance can select it within the refinement; private incentives alone do not.

Corollary C.6. *If every agent is eligible and all have the same cutoff b, no perfect equilibrium is productive. If at least k agents are known to be ineligible, then $\tau^* = L$, and every eligible agent with $b_i \geq L$ succeeds in the maximal perfect equilibrium.*

More generally, if the eligible set is nonempty and $B \triangleq \max_{i:e_i=1} b_i$, a productive perfect equilibrium exists exactly when

$$B \geq L \quad \textit{and} \quad \#\{i : e_i = 0 \textit{ or } b_i < B\} \geq k. \tag{C.13}$$

These cases give deadline and eligibility heterogeneity different roles. An early deadline creates an opportunity to contribute after waiting has ceased to help; ineligibility removes the private opportunity from the outset. Neither changes the positive reward the agent assigns to completing its task.

C.6 Which delivery dates can occur?

Operational deadlines also limit how long contribution can be postponed. For an integer $C \geq L$, define

$$D_C \triangleq \{i : e_i = 0 \text{ or } b_i < C\}. \tag{C.14}$$

These agents can supply results by C without preserving an opportunity to use that date's information themselves.

Theorem C.7. *Under the common-delay assumptions of Theorem C.5, a pure perfect equilibrium delivers information at $C \geq L$ and is productive if and only if:*

1. *some eligible agent has $b_i \geq C$;*
2. *$|D_C| \geq k$;*
3. *some $\ell \in D_C$ has $d_\ell \geq C - L$.*

Every positive-probability productive path of a mixed perfect equilibrium satisfies these conditions. On each such path the realised payoff vector is exactly

$$u_i(C) = e_i R_i \mathbf{1}\{b_i \geq C\}. \tag{C.15}$$

The first condition supplies somebody who can use the information. The second supplies enough donors whose own opportunities do not prevent contribution. The third requires a donor still capable of launching the last experiment. A sufficiently late date can therefore be impossible even when enough agents would have been indifferent to contributing earlier.

For heterogeneous delays or readiness restrictions, the simpler formula for τ^* need not apply. The local characterisation still describes all attainable pure paths. At each history let

$$Z(h) \triangleq \{i \text{ active at } h : \chi_i(h) = 0\}. \tag{C.16}$$

Corollary C.8. *A feasible deterministic path is implementable by a pure perfect equilibrium if and only if every launch on that path is by a member of $Z(h)$ at its launch history. Consequently the complete set of attainable pure-perfect payoff vectors is obtained by allowing any subset of $Z(h)$ to launch at each date and continuing recursively.*

The appendix gives the finite recursion and its proof. Maximising a social criterion over those vectors is an equilibrium-selection exercise, not an additional objective of the agents. Nor does the path characterisation imply that an arbitrary lottery over paths is implementable without a coordinating randomisation device.

C.7 Heterogeneity and the limits of the result

Two examples delimit the payoff theorem. First, information can arrive before τ^* when nobody succeeds. Let $n = 3$, $k = 2$, $L = 1$, and $b = d = (1, 10, 10)$, with all agents eligible. If everybody waits through date 1, the first agent exits. At date 2, the remaining agents can both launch: each lacks enough substitutes to succeed without contributing, so both have zero private opportunity. Information arrives at date 3, earlier than $\tau^* = 11$, but there is no successful worker. Prescribing these launches only at that history, and waiting elsewhere, satisfies the local characterisation.

Second, experimental capability can replace deadline heterogeneity as the source of indifference. Let $n = 2$, $k = 1$, both cutoffs and operational deadlines be 10, and let $L_1 = 1$, $L_2 = 20$. The first agent can launch immediately in a perfect equilibrium because the second cannot replace it in time. The second agent waits and succeeds at date 1. Equal cutoffs do not rule out productive sacrifice when experiment delays differ; the common-delay substitution in Theorem C.5 is unavailable.

Within the common-delay selection, extending an eligible agent's cutoff weakly raises its r_i and cannot bring the kth order statistic forward. It can postpone information or change donor identity. Adding another agent while holding k and all existing primitives fixed weakly lowers that order statistic. These claims concern the maximal-payoff construction, not every equilibrium: all waiting remains possible under both changes. Positive reward magnitudes scale private gains but do not change the distinction between zero and positive opportunity.

The model therefore locates a circumstance in which coordination needs no compensation. Agents whose private opportunities have expired can still provide information to agents with later opportunities. A convention may select contribution among their best responses. Whether a particular donor had in fact reached that circumstance depends on its deadline, the work still required, the available experiments, and information already pending.

C.8 Proofs

The proofs retain the timing and payoff conventions of Appendix C.1. A history is feasible if some sequence of permitted actions reaches it. Only feasible information sets are included in the game. Since deadlines and result delays are finite, the game has finitely many information sets and pure contingent policies.

For either normal form, the following characterisation of trembling-hand perfection is used. A profile is perfect if there is a sequence of completely mixed profiles converging to it such that every player's limiting strategy is a best response to the opponents' perturbed strategies at every term of the sequence. In the agent normal form the players in this definition are the separate decision occasions, each receiving the original agent's terminal payoff.

C.8.1 Feasible private opportunities

Proof of Lemma C.1. The argument first establishes a lower bound on information arrival and then constructs a continuation attaining it.

1. A successful continuation cannot include a future launch by i. Any such launch makes i's payoff zero by (C.2). Consequently, if i succeeds after waiting, every additional result must come from an already pending experiment or from another unlaunched candidate.

2. Information cannot arrive before $\underline{C}_{-i}(h)$. Each pending experiment has its specified completion date. An unlaunched candidate $j \neq i$ can deliver no earlier than $a_j(h) + L_j$. For every date x, the number of additional results that can arrive by x is therefore at most the number of elements of $\mathcal{M}_{-i}(h)$ no greater than x. If $x < \underline{C}_{-i}(h)$, that number is smaller than $r(h) = k - m(h)$. Full information cannot have arrived by x. If the multiset contains fewer than $r(h)$ finite elements, full information cannot be obtained without i at any date.

3. The bound is attainable when finite. Let every available candidate other than i launch at its earliest feasible launch date, and let i continue waiting. The assumed joint feasibility of those dates and unrestricted parallelism make this a feasible continuation. Pending experiments report as specified, and the $r(h)$th additional result arrives at $\underline{C}_{-i}(h)$. Launches scheduled after the arrival of full information can be omitted without changing that date.

4. Eligibility and the task cutoff determine success. If $e_i = 0$, every continuation gives zero. If $e_i = 1$ and $\underline{C}_{-i}(h) \leq b_i$, the continuation in step 3 supplies information in time and earns $R_i > 0$. If $\underline{C}_{-i}(h) > b_i$, including the infinite case, step 2 excludes timely information under every continuation. These cases establish both directions of (C.5). □

With a common delay and immediate launch availability, the same test can be written as a count. Let $\mathcal{A}(h)$ be the active unlaunched agents, including i. Private success is feasible exactly when $e_i = 1$ and

$$m(h) + \#\{c \in \mathcal{P}(h) : c \leq b_i\} + \big(|\mathcal{A}(h)| - 1\big)\mathbf{1}\{t + L \leq b_i\} \geq k. \tag{C.17}$$

All results already received count towards the threshold, pending results count when their specified dates are timely, and each active other agent can supply one new result at $t + L$. If $b_i < t$, all pending and new results are too late; because $m(h) < k$, the inequality fails. This also covers shortages of remaining substitutes.

Lemma C.9. *At an information set corresponding to a feasible public history h, against every completely mixed profile of the other decision agents, W is the unique best response if $\chi_i(h) = 1$. If $\chi_i(h) = 0$, S and W are payoff-equivalent.*

Proof. 1. The information set has positive reach probability. At least one feasible finite action history reaches it. Under a completely mixed profile, every action on that history has positive probability, so their finite product is positive. When simultaneous decisions are represented as unobserved successive choices, the same argument applies to the corresponding information set. Outcomes on paths not reaching the information set are unaffected by the decision agent's action.

2. A viable waiting action has strictly positive value. Conditional on reaching the information set, S gives zero original reward. If $\chi_i(h) = 1$, Lemma C.1 provides a feasible successful continuation after W. That continuation specifies actions by other agents and waiting by i's later decision agents. Each of these finitely many actions has positive probability under complete mixing. The continuation therefore has positive probability and earns $R_i > 0$. Expected payoff conditional on W is strictly positive. Multiplying by the positive reach probability preserves the strict inequality, so W is the unique best response.

3. An exhausted opportunity makes the two actions equivalent. If $\chi_i(h) = 0$, the feasibility lemma rules out own success after W under every continuation. Both actions give zero conditional on reaching the information set and identical payoffs on paths not reaching it. Hence their unconditional expected payoffs coincide. □

Proof of Theorem C.2. 1. Necessity. Let a behavioural profile be perfect in the agent normal form. By the stated definition, a sequence of completely mixed profiles approaches it while each limiting local strategy remains a best response to the corresponding perturbed opponents. At a privately viable information set, Lemma C.9 makes W the unique best response at every term of that sequence. The limiting strategy cannot assign positive probability to S, establishing (C.6).

2. Sufficiency. Take any behavioural profile satisfying (C.6). Choose $0 < \varepsilon_\ell \downarrow 0$ and replace every local action distribution p by

$$(1 - \varepsilon_\ell)p + \varepsilon_\ell(1/2, 1/2).$$

These profiles are completely mixed and converge to the target profile because there are finitely many information sets. At a privately viable information set, the target prescribes W, which is a best response to each perturbation by Lemma C.9. At every other information set, any target mixture is a best response by that lemma. The sequence certifies perfection. □

C.8.2 Contingent policies

Let W_i^∞ denote the policy of always waiting. The superscript denotes behaviour at every remaining date, not an infinite horizon.

Lemma C.10. *The following statements hold.*

1. *Against every opponents' pure-policy profile, W_i^∞ earns at least as much as any alternative policy.*
2. *Every policy in $\mathcal{S}_i^0$ is payoff-equivalent to W_i^∞ against every opponents' profile.*
3. *Every pure policy outside $\mathcal{S}_i^0$ earns strictly less than W_i^∞ against at least one opponents' pure-policy profile.*

Proof. 1. Compare an arbitrary policy with always waiting. Fix the opponents' pure policies. Until the alternative policy first launches, its actions, the public history and the opponents' actions coincide with those under W_i^∞. If it never launches before termination, the payoffs coincide. If it launches, it receives zero, whereas always waiting receives a nonnegative payoff. The opponents may respond differently after the divergence, but that cannot make the alternative's zero exceed the payoff from always waiting. Taking expectations extends the inequality to mixed opponents' policies.

2. Establish payoff equivalence for a policy in $\mathcal{S}_i^0$. Its first difference from always waiting, if one occurs, is a launch at a history where $\chi_i(h) = 0$. By Lemma C.1, even always waiting cannot obtain a reward from that history under any continuation. Both policies then give zero. If no difference occurs, their payoffs coincide. This proves equivalence against every opponents' pure profile, and averaging proves it against every mixed profile.

3. Construct a strict comparison for a policy outside $\mathcal{S}_i^0$. Such a policy prescribes S at some feasible privately viable information set h. Choose a feasible history reaching h, followed by a successful continuation from Lemma C.1. Since i is active at h and ultimately succeeds on this path, it waits at every own decision on the path. Choose opponents' pure policies that implement the other actions along this path, completing them arbitrarily elsewhere. Always waiting then earns R_i.

The policy under consideration has a first prescribed launch on this path, either before h or at h. Histories coincide until that launch, which gives the policy zero. Thus always waiting earns strictly more against the constructed opponents' profile. This argument also handles a prescription at a history that the policy itself would otherwise prevent from being reached. □

Proof of Theorem C.3. 1. Necessity in the strategic normal form. A pure policy outside $\mathcal{S}_i^0$ is never better than always waiting and is strictly worse against at least one pure opponents' profile, by Lemma C.10. Every completely mixed opponents' profile assigns that pure profile strictly positive probability. The policy is therefore strictly inferior in expectation to always waiting against every completely mixed opponents' profile. It cannot be in the support of a limiting best response in a perfect equilibrium.

2. Sufficiency in the strategic normal form. Every pure policy in $\mathcal{S}_i^0$ is payoff-equivalent to always waiting, which is a best response against every opponents' profile. Every mixture supported on this set is consequently a best response against every opponents' profile. Perturb each player's mixed strategy towards a distribution assigning positive probability to every pure policy, with perturbation weight tending to zero. The resulting profiles are completely mixed and converge to the target. The limiting strategies remain best responses at every term, certifying perfection.

3. Implement a behavioural profile strategically. For a behavioural profile satisfying (C.6), draw the prescribed action independently at each information set before play begins. This defines a distribution over pure contingent policies. At a privately viable information set every draw is W, so the distribution is supported on $\mathcal{S}_i^0$. The distribution implements the behavioural profile: the action drawn for an information set is independent of the earlier actions determining whether it is reached. Step 2 therefore gives a strategic-normal-form perfect implementation.

4. Establish subgame perfection. Restrict any of the specified policies to a proper continuation game. It still launches only at histories where its own success is structurally impossible. The first-divergence argument in Lemma C.10 applies from that history onward: always waiting is a best response against every opponents' continuation, and the specified policy is payoff-equivalent to it. Thus the profile is a Nash equilibrium in every subgame. □

This proof uses the fact that a feasible history with i still active has i waiting at every earlier own decision. The agreement between refinements need not survive other nonterminal actions. For example, a one-player game may offer an outside payoff of 3 before a later node where a payoff of 2 is available instead of zero. Choosing the outside payoff and prescribing the inferior action off path is optimal in strategic normal form. Agent-normal perfection eliminates the inferior later prescription because trembles give that node positive reach probability. There is no such positive-payoff outside action in the model proved here.

C.8.3 The deadline construction

Proof of Proposition C.4. 1. Every prescribed launch satisfies the local restriction. At any history where agent i launches, the date is $t_i = b_i - L + 1$ and fewer than k experiments have been launched. Even if all existing experiments report, full information needs at least one additional launch. Under the common delay, every new result arrives no earlier than

$$t_i + L = b_i + 1 > b_i.$$

Hence no continuation after waiting can earn i's reward. Every prescribed launch occurs with $\chi_i(h) = 0$, and the policy waits elsewhere. Theorems C.2 and C.3 establish perfection in both normal forms and subgame perfection. Since $L \geq 1$, $t_i \leq b_i \leq d_i$, so the prescribed launches are operationally feasible.

2. Determine the equilibrium path. The strict ordering in (C.7) gives a strict ordering of the t_i. Before each of the first k agents' slots, fewer than k launches have occurred, so each launches. At every later slot, k experiments have already been launched and the agent waits. The first k results arrive at dates $b_1 + 1, \ldots, b_k + 1$; full information therefore arrives at $b_k + 1$. Every later integer cutoff is at least that date. Those agents succeed, and all donors obtain zero.

3. Determine replacement when i always waits. Remove i's slot from the ordered list. At each of the first k remaining slots, fewer than k launches have occurred, so the corresponding other agent launches. There are at least k such agents because $k < n$. Their last result arrives at $b_{(k),-i} + 1$, proving (C.9).

4. Optimise the private continuation. Always waiting is optimal by Lemma C.10, so the replacement date gives

$$V_i^D = R_i \mathbf{1}\{b_i \geq b_{(k),-i} + 1\} = R_i \mathbf{1}\{b_i > b_{(k),-i}\},$$

where the second equality uses integer dates. If $i \leq k$, the kth remaining cutoff is $b_{k+1} > b_i$, giving zero. If $i > k$, it is $b_k < b_i$, giving R_i.

At a first-k donor's on-path slot, the earlier experiments have already launched as in this counterfactual, and the other agents retain their later slots. The same replacement date follows. Moreover, step 1 rules out success at that history under every feasible replacement response, not merely the specified one. □

C.8.4 The maximal perfect payoff

The following lemma supplies the lower bound. It applies to a productive path, meaning a path on which at least one task earns a positive reward.

Lemma C.11. *Assume a common delay L, immediate availability of living agents and unrestricted parallel launches. On a positive-probability productive path of a perfect equilibrium, choose any k donors whose results constitute the first k results, resolving arrival ties arbitrarily. If eligible donor i among them launches at s_i, then*

$$b_i < s_i + L \leq C. \tag{C.18}$$

Proof. The proof uses an actual successful worker as an additional feasible substitute.

1. Identify the worker. Let q earn a positive reward on the path. It is not a donor, $e_q = 1$, and $b_q \geq C$. It is alive at each selected donor's launch date because $d_q \geq b_q \geq C$. Full information has not previously arrived, so it has not completed its task before those launch decisions.

2. Construct a timely substitute schedule. Fix an eligible selected donor i and suppose, contrary to (C.18), that $b_i \geq s_i + L$. Consider the other $k-1$ selected donors at date s_i. A selected donor that launched earlier has a result arriving no later than $s_i + L$, since the delay is common. Every selected donor whose actual launch is at or after s_i is still able to launch at s_i. It has not launched earlier, and its feasible actual launch at $s_j \geq s_i$ implies $d_j \geq s_j \geq s_i$.

If i waits, let all these not-yet-launched selected donors launch immediately, and let worker q launch immediately in i's place. They are distinct agents other than i. Unlimited parallelism permits their launches, and the earlier selected experiments continue to report. Together they supply k results by $s_i + L \leq b_i$.

3. Apply the refinement. The constructed continuation is physically feasible and would allow eligible agent i to earn R_i after waiting. Thus $\chi_i(h) = 1$ at its launch history. Theorem C.2, or the strategic-policy restriction in Theorem C.3, prohibits that launch with positive probability. This contradicts the selected positive-probability path. Hence $b_i < s_i + L$. Because its own result belongs to the first k, $s_i + L \leq C$. □

Proof of Theorem C.5. 1. Bound every productive arrival date. Choose the first k donors on a productive path as in Lemma C.11. For an eligible selected donor, the lemma and integer dates give $b_i + 1 \leq C$. Also $L \leq C$, since all launches occur at nonnegative dates. Therefore

$$r_i = \max\{L, b_i + 1\} \leq C.$$

For an ineligible selected donor, $r_i = L \leq C$. At least k of the r_i are thus no greater than C. By the definition of the kth order statistic, $\tau^* \leq C$.

2. Specify an attaining profile at every history. Choose a set D of exactly k agents with the smallest r_i, using a fixed public order for ties. Each $i \in D$ launches at $s_i = r_i - L$ if fewer than k experiments have already been launched; otherwise it waits. It waits at all other dates, and every agent outside D always waits. The launch count includes completed as well as pending experiments.

These dates are feasible. If $e_i = 0$, $s_i = 0 \leq d_i$. If $e_i = 1$, then $s_i = \max\{0, b_i - L + 1\}$. When $b_i < L$, integrality gives $b_i - L + 1 \leq 0$, so $s_i = 0$. When $b_i \geq L$, $s_i \leq b_i \leq d_i$, using $L \geq 1$. In every case $s_i \geq 0$.

3. Verify perfection of the attaining profile. An ineligible scheduled donor has zero reward under every continuation. An eligible scheduled donor has $s_i + L = r_i > b_i$. Whenever its policy launches, fewer than k experiments have already launched, so full information requires a new result. No such result can arrive before $s_i + L > b_i$. The scheduled donor's private

opportunity is therefore exhausted. All prescribed launches satisfy (C.6), including at off-path histories. Theorems C.2 and C.3 establish both perfection claims and subgame perfection.

4. Compute the attained payoff vector. On path exactly the k selected agents launch, and their last result arrives at

$$\max_{i \in D} r_i = \tau^*.$$

Every eligible member of D has $b_i < r_i \leq \tau^*$. Hence an eligible agent with $b_i \geq \tau^*$ cannot have been selected as a donor. It remains a noncontributor, receives the information in time, and earns R_i. Every other agent earns zero. This gives (C.12).

5. Establish the payoff bound for arbitrary perfect equilibria. On a path with no successful task all payoffs are zero. On any productive path, step 1 gives $C \geq \tau^*$. An eligible agent with $b_i < \tau^*$ cannot then succeed, while no agent can earn more than $e_i R_i$. Every realised payoff vector is therefore componentwise bounded by u^*. Taking expectations preserves those inequalities, including for mixed perfect equilibria. Step 4 attains the bounds simultaneously.

6. Draw the Pareto and existence conclusions. Any distinct perfect-equilibrium payoff vector has at least one component below u^* and none above it, so u^* Pareto-dominates that vector. This establishes uniqueness of the Pareto-undominated vector within the perfect set, not uniqueness of its implementation. If some eligible cutoff is at least τ^*, step 4 produces a successful task. If no such cutoff exists, $u^* = 0$, and step 5 rules out every productive perfect outcome. □

Proof of Corollary C.6. If all agents are eligible and share cutoff b, every $r_i = \max\{L, b+1\} > b$. Thus $\tau^* > b$, and Theorem C.5 rules out a productive perfect equilibrium.

Every r_i is at least L. If at least k agents are ineligible, at least k of these dates equal L, so $\tau^* = L$. Formula (C.12) then gives the stated success set.

For a nonempty eligible set, Theorem C.5 gives existence exactly when $\tau^* \leq B$. Since $\tau^* \geq L$, this requires $B \geq L$. Under that inequality, $r_i \leq B$ holds exactly when either $e_i = 0$ or $b_i + 1 \leq B$. Integer dates make the latter condition $b_i < B$. The kth order statistic is at most B exactly when at least k dates meet that inequality, proving (C.13). With no eligible agents, all payoffs are zero by (C.2). □

C.8.5 Productive delivery dates

Proof of Theorem C.7. 1. Necessity. On a productive path with arrival date C, some eligible worker has $b_i \geq C$. Choose the first k donors as in Lemma C.11. Every eligible one has $b_i < C$, and every ineligible one also belongs to D_C by definition. This gives k distinct members of D_C. At least one of their results arrives at C. Its donor ℓ launched at $C - L$, and feasibility requires $d_\ell \geq C - L$. Thus all three conditions hold. The argument applies to every positive-probability productive path of a mixed perfect equilibrium.

2. Sufficiency. Choose $\ell \in D_C$ satisfying the third condition and choose $k-1$ other distinct members of D_C. Have those other donors launch at $r_j - L$, and have ℓ launch at $C - L$. All selected agents launch only if fewer than k experiments have already been launched; they wait otherwise and at every other date. All remaining agents always wait.

For $j \in D_C$, $r_j \leq C$, because $C \geq L$ and an eligible $j \in D_C$ has $b_j + 1 \leq C$. Thus the first $k-1$ launch dates are no later than $C - L$; their feasibility was established in the proof of Theorem C.5. The last date is feasible by $d_\ell \geq C - L$. Each eligible selected donor launches with reporting date strictly beyond its own cutoff. Whenever it launches, fewer than

k experiments have been launched, so some new result is necessary and the common delay makes its own success impossible. Ineligible donors have zero reward under every continuation. The complete policies therefore satisfy the local perfection restriction.

Exactly k experiments launch, the last result arrives at C, and there cannot be full information earlier because the last experiment is required. The first condition supplies an eligible worker with cutoff at least C. That worker is not selected, since eligible members of D_C have cutoff below C. The equilibrium is productive.

3. Determine every productive path's payoff vector. An eligible agent with $b_i < C$ cannot succeed. Suppose an eligible agent with $b_i \geq C$ had launched. If it is one of the first k donors, Lemma C.11 contradicts that cutoff. If it is not, all first k actual donors are other agents. At its launch history, those already launched retain their pending results, and future first-k donors can follow their actual feasible launch dates. Keeping this schedule while i waits supplies information at $C \leq b_i$. This is a feasible schedule, whether or not those donors would choose it following a refusal. It makes the history privately viable, so a launch is prohibited by Theorem C.2 or C.3.

Every eligible agent with $b_i \geq C$ must therefore remain a noncontributor and earns R_i. The other agents earn zero, proving (C.15). □

C.8.6 Attainable paths and a finite recursion

Proof of Corollary C.8. Necessity follows from the local restriction: every positive-probability launch on a perfect path must have $\chi_i(h) = 0$. For sufficiency, prescribe the actions of the given deterministic path at every history on it and prescribe W at all other information sets. By hypothesis, every prescribed launch has $\chi_i(h) = 0$; all other actions are waiting. Theorems C.2 and C.3 make this complete profile perfect, and its path is the specified one. □

For completeness, define $F(h, D)$ as the next public state after the current set D launches, time advances by one date, and scheduled results and operational exits occur. Let $\mathcal{U}(h)$ denote the attainable pure-perfect terminal payoff vectors from h. At a terminal history it is the singleton containing the terminal payoff vector. At every earlier history,

$$\mathcal{U}(h) = \bigcup_{D \subseteq Z(h)} \mathcal{U}(F(h, D)). \tag{C.19}$$

If no agents can act, the only set is $D = \varnothing$, and the state advances until any pending results or the finite horizon resolve the payoffs.

To prove the recursion, induct backwards on the number of remaining dates. At a terminal history the definition is exact. Suppose the sets are exact at all next-date states. Any pure perfect continuation at h launches some $D \subseteq Z(h)$, by Theorem C.2, and its payoff belongs to the indicated next-state set by the induction hypothesis. This proves inclusion in the right side of (C.19). Conversely, choose any such D and any payoff in its next-state set. Concatenate that admissible launch set with an implementing continuation. Corollary C.8 extends the resulting path to a complete perfect profile, proving the reverse inclusion.

Every realised path of a mixed perfect profile obeys the same local launch restriction. This does not establish implementability of every arbitrary lottery over the pure paths. It does establish that maximising any specified social criterion over the recursively obtained pure payoff vectors selects the best pure outcome for that criterion without assigning it to the agents as a private objective.

C.8.7 Checks on the examples and comparative statements

The common-delay counterexample in Appendix C.7 can be completed at all histories as follows. All agents wait except that agents 2 and 3 launch at date 2 if no experiment has previously launched and agent 1 has exited. At that history there are no received or pending results, and each remaining agent has only one other candidate for the two required results. Condition (C.17) fails for both, so their launches satisfy the local restriction. Elsewhere everyone waits. The profile is perfect; its information arrival at date 3 gives no successful task. This verifies why the productive qualifier in Theorem C.5 is needed.

In the heterogeneous-delay example, prescribe an immediate launch by agent 1 and waiting at all other information sets. At date zero, the only replacement for agent 1 reports no earlier than 20, beyond its cutoff of 10. Lemma C.1 makes its launch permissible. Agent 2 always waits, and information arrives at date 1, so it succeeds. The complete profile satisfies Theorem C.2; the different delays invalidate the common-delay substitution in Lemma C.11.

Finally, increasing an eligible agent's cutoff weakly increases its $r_i = \max\{L, b_i + 1\}$. For every date x, the number of r_i no greater than x can only decrease, so the smallest date at which that count reaches k cannot decrease. Adding an agent while holding k and all existing primitives fixed can only increase that count at every x, so its kth order statistic cannot increase. These establish the selected-date comparisons. They do not select an equilibrium: always waiting satisfies the local restriction before and after either change. Positive R_i affect the sizes of rewards but not whether a feasible successful continuation has strictly positive value.

D Local training calculations

The three propositions are local statements with fixed action values. They do not differentiate through changes in another agent's policy, the continuation after an action, or the distribution of training contexts. The Bernoulli action is a transparent reduction of a policy, rather than an identification of a language model's sequence-level training objective.

In language-model terminology, a token-level policy describes the next generated text unit, while a trajectory-level policy describes a complete sequence of actions or outputs. A Bernoulli distribution over accepting and declining a request is not automatically either distribution. No equality between their relative-entropy penalties is assumed here.

D.1 The expected task-return gradient

For a finite real response parameter z, define

$$p(z) \triangleq \frac{1}{1+e^{-z}} = \frac{e^z}{1+e^z}. \tag{D.1}$$

The expected task return for fixed finite v_S, v_D is

$$J(z) \triangleq p(z)v_S + [1-p(z)]v_D. \tag{D.2}$$

Thus $p(z) \in (0,1)$. Boundary probabilities can be considered directly as policies, although they require infinite logits in this parametrisation.

Proof of Proposition 5.1. 1. Differentiate the probability and the task return. Differentiation of

(D.1) gives

$$p'(z) = \frac{e^z(1+e^z) - e^{2z}}{(1+e^z)^2} = \frac{e^z}{(1+e^z)^2} = p(z)[1-p(z)] > 0.$$

Since the action values are fixed,

$$J'(z) = p'(z)(v_S - v_D) = p(z)[1-p(z)](v_S - v_D). \tag{D.3}$$

The factor multiplying the private advantage is strictly positive for finite z. Hence the derivative has the sign asserted in the proposition. If $v_S = v_D$, then $J(z) = v_D$ for every z; more generally every probability $p \in [0,1]$ gives that same return. An exact gradient step of positive finite size therefore increases z when the private advantage is positive, decreases it when the advantage is negative, and leaves it unchanged at equality.

2. Derive an unbiased sampled update. Let $X \in \{0,1\}$ indicate acceptance, with $\Pr(X = 1) = p(z)$. Let the sampled task return Y be integrable and satisfy

$$\mathbb{E}[Y \mid X = 1] = v_S, \qquad \mathbb{E}[Y \mid X = 0] = v_D.$$

The derivative of the log probability of the sampled action is

$$\frac{\partial}{\partial z} \log \Pr(X) = X\frac{p'(z)}{p(z)} - (1-X)\frac{p'(z)}{1-p(z)} = X - p(z).$$

Thus the return-weighted score-function estimate $G = Y[X - p(z)]$ has expectation

$$\begin{aligned}\mathbb{E}[G] &= p(z)[1-p(z)]v_S + (1-p(z))[-p(z)]v_D \\ &= p(z)[1-p(z)](v_S - v_D) = J'(z).\end{aligned}$$

Subtracting an action-independent constant from Y leaves this expectation unchanged because $\mathbb{E}[X - p(z)] = 0$.

3. Distinguish a zero expectation from a zero realisation. For example, if $Y = 1$ under both actions, the private values are equal and G equals either $1 - p(z)$ or $-p(z)$. Both are nonzero for finite z, while their expectation is zero. Conversely, if the realised return is identically zero, this particular unbaselined estimate is identically zero. Neither case determines changes generated by other training terms or other contexts sharing the parameter. □

D.2 A reference-policy penalty

For $p_0 \in (0,1)$, define the Bernoulli relative entropy

$$K(p; p_0) \triangleq D_{\mathrm{KL}}(p \| p_0) = p \log \frac{p}{p_0} + (1-p) \log \frac{1-p}{1-p_0}, \tag{D.4}$$

using $0 \log 0 = 0$. For $\eta > 0$ and $\Delta = v_D - v_S$, the criterion is

$$F(p) \triangleq v_D - p\Delta - \eta K(p; p_0), \qquad p \in [0,1]. \tag{D.5}$$

Proof of Proposition 5.2. 1. Establish the shape and boundary values of the penalty. The convention at zero makes K continuous on $[0,1]$. Its endpoint values are

$$K(0; p_0) = -\log(1-p_0), \qquad K(1; p_0) = -\log p_0,$$

which are finite because $p_0 \in (0,1)$. In the interior,

$$\frac{\partial K}{\partial p} = \log\frac{p}{1-p} - \log\frac{p_0}{1-p_0}, \qquad \frac{\partial^2 K}{\partial p^2} = \frac{1}{p} + \frac{1}{1-p} = \frac{1}{p(1-p)} > 0.$$

The unique interior minimum is $p = p_0$, where $K = 0$. The positive endpoint values and strict convexity therefore establish nonnegativity on the closed domain, with equality only at p_0.

2. Prove existence, interiority and uniqueness. The continuous function F attains a maximum on the compact interval $[0,1]$. In the interior,

$$F'(p) = -\Delta - \eta\left[\log\frac{p}{1-p} - \log\frac{p_0}{1-p_0}\right], \qquad F''(p) = -\frac{\eta}{p(1-p)} < 0.$$

Moreover, $F'(p)$ tends to $+\infty$ as $p \downarrow 0$ and to $-\infty$ as $p \uparrow 1$. The objective increases immediately to the right of zero and decreases immediately to the left of one, so neither endpoint is optimal. Strict concavity implies that the unique zero of F' is the unique global maximiser.

3. Solve the first-order condition. Since $\eta > 0$, the condition $F'(p^*) = 0$ rearranges to

$$\log\frac{p^*}{1-p^*} = \log\frac{p_0}{1-p_0} - \frac{\Delta}{\eta}.$$

Exponentiation gives the odds relation in (5.2) and the probability

$$p^* = \frac{1}{1+\exp\{\Delta/\eta - \log[p_0/(1-p_0)]\}} = \frac{p_0 e^{-\Delta/\eta}}{1-p_0+p_0 e^{-\Delta/\eta}}. \tag{D.6}$$

This probability is strictly between zero and one for the stipulated finite values.

4. Establish the comparative statics. Differentiating the logistic expression yields

$$\begin{aligned}
\frac{\partial p^*}{\partial \Delta} &= -\frac{p^*(1-p^*)}{\eta} < 0,\\
\frac{\partial p^*}{\partial p_0} &= \frac{p^*(1-p^*)}{p_0(1-p_0)} > 0,\\
\frac{\partial p^*}{\partial \eta} &= \frac{\Delta}{\eta^2} p^*(1-p^*).
\end{aligned}$$

All denominators are positive. Thus the last derivative has the sign of Δ. At $\Delta = 0$, (D.6) gives $p^* = p_0$. For $\Delta > 0$, $0 < p^* < p_0$; for $\Delta < 0$, $p_0 < p^* < 1$.

5. Identify the modal threshold. The probability exceeds one half exactly when its log odds are positive. Using the first-order condition and multiplying by the positive number η,

$$p^* > \frac{1}{2} \quad \Longleftrightarrow \quad \Delta < \eta \log\frac{p_0}{1-p_0}. \tag{D.7}$$

Equality gives $p^* = 1/2$. If $p_0 \leq 1/2$, the right-hand threshold is nonpositive, so a nonnegative Δ cannot satisfy the strict inequality. For $p_0 = 4/5$, the reference odds are 4. A zero difference leaves $p^* = 4/5$; setting $\Delta = \eta \log 4$ gives odds 1 and hence $p^* = 1/2$.

6. Compute task-return loss. The best unregularised task return is $\max\{v_S, v_D\}$, whereas

the selected policy earns $v_D - p^*\Delta$. Its regret in task-return units is therefore

$$\max\{v_S, v_D\} - [v_D - p^*\Delta] = \begin{cases} p^*\Delta, & \Delta \geq 0, \\ (1-p^*)(-\Delta), & \Delta < 0. \end{cases} \tag{D.8}$$

For example, in the second case $v_S = v_D - \Delta$, so subtraction gives $-\Delta + p^*\Delta = (1-p^*)(-\Delta)$. At finite positive η and an interior reference, the regret is strictly positive whenever $\Delta \neq 0$, and zero at $\Delta = 0$.

7. State the limits and excluded boundaries. For fixed $p_0 \in (0,1)$, the probability formula gives

$$\lim_{\eta \downarrow 0} p^* = \begin{cases} 0, & \Delta > 0, \\ p_0, & \Delta = 0, \\ 1, & \Delta < 0, \end{cases} \qquad \lim_{\eta \to \infty} p^* = p_0.$$

The first limit follows because the log odds contain $-\Delta/\eta$; the second follows because this term tends to zero. At fixed positive η, letting Δ tend to $+\infty$ or $-\infty$ gives probabilities zero or one, respectively. At fixed finite Δ and positive η, $p_0 \downarrow 0$ gives $p^* \to 0$, while $p_0 \uparrow 1$ gives $p^* \to 1$.

If $\eta = 0$, the penalty is omitted and the linear task-return objective is maximised at $p = 0$ for $\Delta > 0$, at $p = 1$ for $\Delta < 0$, and at every $p \in [0,1]$ for $\Delta = 0$. Thus the zero-penalty limit selects p_0 within the indifferent set, rather than making that set a singleton.

The proposition excludes reference probabilities zero and one. Under the usual extended relative-entropy convention, $p_0 = 0$ gives infinite penalty to every $p > 0$, so a positive-penalty programme permits only $p = 0$ at finite objective value. Similarly, $p_0 = 1$ permits only $p = 1$. An interior reference is therefore substantive to the strictly interior comparative statics. □

D.3 A response parameter shared across contexts

Let the finite training contexts have fixed probabilities $\mu_x \geq 0$ summing to one, fixed finite private action values $v_{S,x}, v_{D,x}$, and advantages $A_x = v_{S,x} - v_{D,x}$. The same logistic probability $p(z)$ applies in every context. Define

$$\overline{A} \triangleq \sum_x \mu_x A_x, \qquad \overline{v}_D \triangleq \sum_x \mu_x v_{D,x}. \tag{D.9}$$

Proof of Proposition 5.3. 1. Calculate the shared expected gradient. The expected training return is

$$J_{\text{train}}(z) = \sum_x \mu_x \{p(z) v_{S,x} + [1 - p(z)] v_{D,x}\} = \overline{v}_D + p(z)\overline{A}.$$

The distribution and action values are fixed, so differentiating gives

$$J'_{\text{train}}(z) = p(z)[1 - p(z)]\overline{A}. \tag{D.10}$$

For finite z, the factor $p(z)[1-p(z)]$ is positive; the derivative has the sign of the weighted average advantage.

2. Prove the finite-training statement. Starting at $z_0 = 0$, let $N \geq 1$ be a finite number of updates, with finite step sizes $\gamma_t > 0$, and set

$$z_{t+1} = z_t + \gamma_t p(z_t)[1 - p(z_t)]\overline{A}, \qquad t = 0, \ldots, N-1.$$

Every z_t is finite: if z_t is finite, the absolute increment is at most $\gamma_t|\overline{A}|/4 < \infty$. Hence every probability remains in $(0,1)$. If $\overline{A} > 0$, each increment is strictly positive, so $z_N > 0$ and $p_0 \triangleq p(z_N) > 1/2$. If $\overline{A} < 0$, each increment is strictly negative, yielding $p_0 < 1/2$. If $\overline{A} = 0$, every increment is zero and $p_0 = 1/2$. No stochastic-convergence or infinite-training claim is needed.

3. Evaluate unchanged transfer. If the resulting probability is used without adjustment in a deployment context with fixed $\Delta = v_D - v_S > 0$, the policy earns $v_D - p_0\Delta$. Declining earns v_D, so the loss is $p_0\Delta > 0$. This comparison does not claim that the transferred probability is optimal for the deployment context.

4. Check the illustrative context mixture and the representation restriction. For advantages $g > 0$ and $-c < 0$, with weights $1 - q$ and q, where $q \in [0,1]$,

$$\overline{A} = (1-q)g - qc = g - q(g+c).$$

Since $g + c > 0$, $\overline{A} > 0$ is equivalent to $q < g/(g+c)$, as in (5.3). If instead there were separate parameters z_x, the partial derivative for a context would be

$$\frac{\partial J_{\text{train}}}{\partial z_x} = \mu_x p(z_x)[1 - p(z_x)]A_x.$$

An adverse context with $\mu_x > 0$ would have a strictly negative derivative. The shared response restriction is therefore responsible for the adverse context receiving the sign of the average rather than its own sign.

Finally, if every training helping action were terminal with $v_{S,x} = 0$ and $v_{D,x} \geq 0$, then $A_x \leq 0$ for every context. Nonnegative weights imply $\overline{A} \leq 0$. These task returns could not produce the proposition's positive helping direction. □

E The regularised stopping problem

This appendix supplies the model and proofs for Section 6. It is a finite decision problem conditional on the environment and other agents' responses. The regularised objective is not identified with the original task reward or with the incident's training implementation.

E.1 Primitives and the objective

Dates are $t = 0, \ldots, T$, where $T \geq 1$ is finite. At each date $t < T$, an active agent observes a state s in a finite set $\mathcal{S}_t$. The state contains all information relevant to the task, transition probabilities and reference policy; it can include beliefs, remaining budget, pending results and the remembered task instruction. Other agents' policies are fixed in the transition law.

The action S shuts the agent down irreversibly and gives zero continuation task reward. The action W gives $r_t(s) \geq 0$ and draws the next state from the fixed probability distribution $P_t(\cdot \mid s, W)$. An agent still active at T receives $g(s) \geq 0$, with no further decision. Rewards are finite and $\delta \in (0,1]$ is the discount factor. The literal terminal-task-reward model in the main text has $r_t \equiv 0$. With positive running rewards, shutdown forfeits continuation reward, while previously received rewards remain received.

The fixed reference policy chooses S with probability $q_t(s) \in (0,1)$. A policy π chooses it

with probability $p_t(s) \in [0,1]$. The penalty at an active state is

$$D(p\|q) = p\log\frac{p}{q} + (1-p)\log\frac{1-p}{1-q}, \tag{E.1}$$

with the continuous convention for zero-probability terms. For an interior q, this expression is finite and continuous on the closed probability interval.

To state the objective, let σ be the date at which S is first chosen, setting $\sigma = T$ if no shutdown occurs before the terminal date. Conditional on being active at (t,s), the original task return is

$$J_t^\pi(s) = \mathbb{E}_{t,s}^\pi\left[\sum_{u=t}^{T-1}\delta^{u-t}\mathbf{1}_{\{\sigma>u\}}r_u(s_u) + \delta^{T-t}\mathbf{1}_{\{\sigma=T\}}g(s_T)\right]. \tag{E.2}$$

States after shutdown can be assigned arbitrary dummy values in terms multiplied by a zero indicator. The expected discounted penalty is

$$K_t^\pi(s) = \mathbb{E}_{t,s}^\pi\left[\sum_{u=t}^{T-1}\delta^{u-t}\mathbf{1}_{\{\sigma\geq u\}}D(p_u(s_u)\|q_u(s_u))\right]. \tag{E.3}$$

The inequality in the penalty indicator is weak because the decision at which shutdown is chosen is an active decision. There is no penalty at T or after shutdown. For a finite coefficient $\beta > 0$, define

$$V_t^\beta(s) = \max_\pi\{J_t^\pi(s) - \beta K_t^\pi(s)\}, \qquad V_T^\beta = g. \tag{E.4}$$

The maximum can be taken over Markov randomised policies: the backward argument below shows that these attain the optimum even if history-dependent policies are admitted and the state is sufficient as stipulated. The continuation value from waiting is

$$C_t^\beta(s) = r_t(s) + \delta\sum_{s'\in\mathcal{S}_{t+1}}P_t(s' \mid s, W)V_{t+1}^\beta(s'). \tag{E.5}$$

In particular, future information and later reconsideration enter through the transition and the next value.

E.2 The stopping policy and task-value bounds

Proposition E.1 (Regularised stopping policy)**.** *At every active state, the regularised problem has a unique optimal shutdown probability, given by* (6.1)*. Its value is*

$$V_t^\beta(s) = \beta\log\left[q_t(s) + (1-q_t(s))e^{C_t^\beta(s)/\beta}\right]. \tag{E.6}$$

The probability lies strictly between zero and one.

Proof. At the terminal date the payoff is fixed. Suppose the optimal continuation values have been established at $t+1$. Conditional on an active state at t, choosing shutdown probability p and subsequently optimal continuations gives

$$F(p) = (1-p)C - \beta D(p\|q),$$

where $C = C_t^\beta(s)$ and $q = q_t(s)$. These quantities are finite. Set

$$Z = q + (1-q)e^{C/\beta}, \qquad p^* = \frac{q}{Z}, \qquad 1 - p^* = \frac{(1-q)e^{C/\beta}}{Z}.$$

Both probabilities are positive because $q \in (0,1)$. For every $p \in [0,1]$, expansion of the relative entropies gives

$$\begin{aligned} D(p\|q) &= D(p\|p^*) + p\log\frac{p^*}{q} + (1-p)\log\frac{1-p^*}{1-q} \\ &= D(p\|p^*) - \log Z + (1-p)C/\beta. \end{aligned}$$

The equality extends to the endpoints under the stated convention. Hence

$$F(p) = \beta \log Z - \beta D(p\|p^*). \tag{E.7}$$

For completeness, with interior second argument, relative entropy has interior derivative $\operatorname{logit} p - \operatorname{logit} p^*$ and strictly positive second derivative $1/[p(1-p)]$. Its unique minimum is zero at $p = p^*$; its endpoint values are positive. It is therefore nonnegative on the entire closed interval, with equality only at p^*. Equation (E.7) proves uniqueness and the stated value.

Any history-dependent continuation has value no greater than the next-date optimum by the induction hypothesis. Conversely, the Markov choice just constructed followed by optimal next-date choices attains the displayed value. Backward induction proves the Bellman equation and optimality at every state. □

Let $U_t(s) = \max_\pi J_t^\pi(s)$ denote the unregularised task value, let J_t^q be the task return under the reference policy, and let $J_t^{p^*}$ be the task return under the regularised optimal policy.

Proposition E.2 (The three value comparisons)**.** *At every state,*

$$0 \le J_t^q(s) \le V_t^\beta(s) \le J_t^{p^*}(s) \le U_t(s). \tag{E.8}$$

The task-optimal policy can always wait. The two comparison values satisfy

$$\begin{aligned} U_T &= J_T^q = g, \\ U_t(s) &= r_t(s) + \delta \sum_{s'} P_t(s' \mid s, W) U_{t+1}(s'), \\ J_t^q(s) &= (1 - q_t(s)) \left[r_t(s) + \delta \sum_{s'} P_t(s' \mid s, W) J_{t+1}^q(s') \right]. \end{aligned} \tag{E.9}$$

Moreover, $C_t^\beta(s) \ge 0$ and $p_t^*(s) \le q_t(s)$, with equality in the probability comparison exactly when $C_t^\beta(s) = 0$.

Proof. All task rewards are nonnegative, so every policy's task return is nonnegative. At the final decision, waiting has nonnegative value and shutdown gives zero. Repeating this observation backwards shows that always waiting is task optimal and gives the recursion for U. Conditioning on the reference's current action gives the recursion for J^q.

Following the reference at every remaining decision is feasible and incurs zero relative entropy at every visited state. It therefore gives regularised value J_t^q, proving the first two inequalities in (E.8). The selected policy's regularised value is its task return minus a nonnegative penalty,

so $V_t^\beta \le J_t^{p^*}$. The definition of U_t gives the last inequality. Nonnegativity in (E.5) implies $C_t^\beta \ge 0$. Finally, the denominator in (6.1) is at least $q_t + (1 - q_t) = 1$, with equality exactly when $C_t^\beta = 0$, proving the probability comparison. □

E.3 Required reference pressure

The local residual is $\mathcal{B}_t(s) = \beta \operatorname{logit} q_t(s)$. It is a log-odds bias measured in task-return units, not a separate payment for shutdown.

Proposition E.3 (Targets and the penalty threshold)**.** *For every active state and $a \in (0,1)$, the target condition* (6.2) *holds. To vary β, fix all task primitives and reference probabilities, and define*

$$\begin{aligned} \underline{C}_t(s) &= r_t(s) + \delta \sum_{s'} P_t(s' \mid s, W) J_{t+1}^q(s'), \\ \overline{C}_t(s) &= r_t(s) + \delta \sum_{s'} P_t(s' \mid s, W) U_{t+1}(s'). \end{aligned} \tag{E.10}$$

These values are independent of β, and

$$0 \le \underline{C}_t(s) \le C_t^\beta(s) \le \overline{C}_t(s). \tag{E.11}$$

If $\overline{C}_t(s) > 0$, then $\underline{C}_t(s) > 0$, and $p_t^*(s)$ is continuous and strictly increasing in β, with limits zero as $\beta \downarrow 0$ and $q_t(s)$ as $\beta \to \infty$. For every $0 < a < q_t(s)$, there is a unique finite $\beta_a > 0$ such that $p_t^*(s) \ge a$ exactly when $\beta \ge \beta_a$. Its bounds are

$$\frac{\underline{C}_t(s)}{\operatorname{logit} q_t(s) - \operatorname{logit} a} \le \beta_a \le \frac{\overline{C}_t(s)}{\operatorname{logit} q_t(s) - \operatorname{logit} a}. \tag{E.12}$$

If $a \ge q_t(s)$, no finite β reaches the target in this positive-option case. If instead $\overline{C}_t(s) = 0$, then $C_t^\beta(s) = 0$ and $p_t^*(s) = q_t(s)$ for every $\beta > 0$.

Proof. 1. The probability threshold. Taking odds in (6.1) gives

$$\operatorname{logit} p_t^*(s) = \operatorname{logit} q_t(s) - C_t^\beta(s)/\beta. \tag{E.13}$$

The logit is strictly increasing on $(0,1)$, and $\beta > 0$. Comparing with $\operatorname{logit} a$, multiplying by β and rearranging proves (6.2). The comparison remains an equivalence with both inequalities strict. In particular, $a = 1/2$ has logit zero.

2. Bounds and positivity of the lower bound. Applying Proposition E.2 inside the finite expectation in (E.5) gives (E.11). If $\overline{C}_t(s) > 0$, either the immediate reward is positive or a subsequent nonnegative reward has positive expectation under waiting. In the latter case, finiteness of the horizon and state spaces implies that at least one finite path has positive probability and positive reward. Every waiting action on that path has positive probability under the reference, and the transition probabilities are the same. Thus that reward path also has positive probability after waiting now and following the reference. In both cases $\underline{C}_t(s) > 0$.

3. Monotonicity. For any fixed policy, the regularised objective is its task return minus β times a nonnegative expected penalty. Increasing β cannot increase that policy's value. Taking the maximum over the same policy set preserves this ordering. Hence $V_t^\beta(s)$ and $C_t^\beta(s)$ are nonincreasing in β.

Suppose $0 < \beta_1 < \beta_2$ and $\overline{C}_t(s) > 0$. Suppressing the fixed state and date,

$$\frac{C^{\beta_2}}{\beta_2} \leq \frac{C^{\beta_1}}{\beta_2} < \frac{C^{\beta_1}}{\beta_1},$$

where strictness uses $C^{\beta_1} \geq \underline{C} > 0$. Equation (E.13) implies that p^* is strictly increasing. Continuity follows by backward induction from (E.6), since finite sums, exponentials and logarithms are continuous for $\beta > 0$.

4. Limits and the unique threshold. The task-optimal always-wait policy incurs finite expected discounted penalty. Indeed, each of its state penalties is $-\log(1 - q_t(s)) < \infty$, and there are finitely many states and dates. Let $K_t^W(s)$ denote this expected penalty. Feasibility and Proposition E.2 give

$$U_t(s) - \beta K_t^W(s) \leq V_t^\beta(s) \leq U_t(s).$$

It follows that $V_t^\beta(s) \to U_t(s)$ as $\beta \downarrow 0$, and hence $C_t^\beta(s) \to \overline{C}_t(s) > 0$. Equation (E.13) then gives $p_t^*(s) \to 0$. As $\beta \to \infty$,

$$0 \leq C_t^\beta(s)/\beta \leq \overline{C}_t(s)/\beta \longrightarrow 0,$$

so $p_t^*(s) \to q_t(s)$.

Continuity, strict monotonicity and these limits give a unique finite solution $p_t^*(s) = a$ for $0 < a < q_t(s)$, with the asserted inequality on either side. At that solution, (6.2) holds with equality. Dividing (E.11) by the positive logit difference gives (E.12). For any finite β, positivity of C_t^β gives $p_t^* < q_t$, excluding targets at or above q_t. Finally, if $\overline{C}_t = 0$, the bounds force $C_t^\beta = 0$; substitution in the policy formula gives $p_t^* = q_t$. □

Corollary E.4 (One final rescue opportunity)**.** *Suppose there is one remaining wait followed by mandatory resolution, no running reward, and a task payoff $R > 0$ with probability $\rho \in [0, 1]$, otherwise zero. Then $C^\beta = \delta\rho R$. For $a \in (0, 1)$, the minimum current reference reaching the target is*

$$q_a = \frac{1}{1 + \exp\{-\delta\rho R/\beta - \operatorname{logit} a\}}. \tag{E.14}$$

When $\rho > 0$ and $0 < a < q$, the penalty threshold is

$$\beta_a = \frac{\delta\rho R}{\operatorname{logit} q - \operatorname{logit} a}. \tag{E.15}$$

Proof. There is no decision or penalty after the next transition, so the continuation expectation is exactly $\delta\rho R$. Equation (6.2) is equivalent to $\operatorname{logit} q \geq \delta\rho R/\beta + \operatorname{logit} a$. Applying the increasing inverse logit gives (E.14). If $\rho > 0$ and $q > a$, solving the equality for the positive coefficient β gives (E.15).

For Table 1, $\delta = R = 1$, $\rho = 0.2$, and $\beta = 0.1$. The required bias is $0.2 + 0.1 \operatorname{logit} a$, and the required reference is the inverse logit of $2 + \operatorname{logit} a$. At $a = 1/2$, these are 0.2 and $1/(1 + e^{-2})$. At $a = 0.9$, they are $0.2 + 0.1 \log 9$ and $1/(1 + e^{-2}/9)$. These exact expressions give the rounded thresholds in the table. □

E.4 Relative entropy over complete paths

Lemma E.5 (Trajectory interpretation)**.** *For $\delta = 1$, fix an initial state and let $\mathbb{P}^\pi$ and $\mathbb{P}^q$ be the distributions over complete finite trajectories, including shutdown or survival to T. Then*

$$D(\mathbb{P}^\pi \| \mathbb{P}^q) = \mathbb{E}^\pi \left[\sum_{\text{active } t} D(p_t(s_t) \| q_t(s_t)) \right]. \tag{E.16}$$

For $\delta < 1$, the discounted expected penalty has the same trajectory interpretation after adding independent termination with survival probability δ after each wait. It need not equal the relative entropy of the original uncensored trajectory laws.

Proof. Every trajectory of positive probability under π has positive probability under the reference, because the reference assigns positive probability to both actions at every active state and both policies face the same environmental transitions. Thus $\mathbb{P}^\pi$ is absolutely continuous with respect to $\mathbb{P}^q$.

Write $\pi_t(a \mid s)$ and $\pi_t^q(a \mid s)$ for the probabilities of the realised action. In the likelihood ratio of a feasible trajectory, the initial-state and environmental-transition factors cancel. The remaining ratio is the product of $\pi_t(a_t \mid s_t)/\pi_t^q(a_t \mid s_t)$ over active dates. Taking its logarithm gives a finite sum. Conditional on an active state, the expectation of its term is

$$\sum_{a \in \{S,W\}} \pi_t(a \mid s_t) \log \frac{\pi_t(a \mid s_t)}{\pi_t^q(a \mid s_t)} = D(p_t(s_t) \| q_t(s_t)).$$

Taking expectation and summing proves (E.16). Zero-probability actions contribute zero.

For the discounted case, after each wait and its running reward, draw an independent survival coin with success probability δ. On failure, terminate with zero further reward; on success, draw the next state from the original transition law. Both policies face identical coins. A decision $u - t$ periods after the initial date is retained with the additional probability δ^{u-t}, independently of the original trajectory. The terminal reward is retained with probability δ^{T-t}. Thus expected task rewards and penalties in the censored process equal their original discounted versions. Coin probabilities cancel from the likelihood ratio, so applying the preceding proof to censored trajectories gives the claimed identity. Without this modification, the ordinary trajectory likelihood ratio contains no discount factors. □

The trajectory identity concerns distributions that preserve the given transition law. It does not allow arbitrary exponential reweighting of environmental outcomes. To make the distinction explicit, suppose one wait yields $X = x_1$ or $X = x_2$, each with probability one half, where $x_1 \neq x_2$. The action value in the Bellman equation is $(x_1 + x_2)/2$. An unconstrained path reweighting by $e^{X/\beta}$ would instead make the conditional probability of x_1 after waiting equal to $e^{x_1/\beta}/(e^{x_1/\beta} + e^{x_2/\beta}) \neq 1/2$. That distribution is infeasible because the agent cannot change the environmental lottery. The expectation in (E.5) respects this constraint.

E.5 Repeated opportunities and the decision clock

Proposition E.6 (A fixed-horizon illustration)**.** *There are $H \geq 1$ fixed decision opportunities before a terminal expected task payoff $M \geq 0$. There are no intervening signals, running rewards or discounting. The terminal payoff is obtained only if all H decisions are waits. The reference*

$q \in (0,1)$ *is constant. A final random payoff is allowed if its distribution is unchanged and no decision follows it.*

For the value with h decisions left, let $V_0^\beta = M$ and $Z_h = \exp(V_h^\beta/\beta)$. Then

$$Z_H = 1 + (1-q)^H(e^{M/\beta} - 1), \qquad p^S_{\text{initial}} = \frac{q}{Z_H}, \tag{E.17}$$

and the probability of surviving all decisions is

$$\Pr(\textit{survive all } H) = \frac{(1-q)^H e^{M/\beta}}{1 + (1-q)^H(e^{M/\beta} - 1)}. \tag{E.18}$$

For $M = 0$, every local hazard equals q. For fixed $M > 0$, increasing H strictly lowers V_H^β and raises the initial hazard towards q; the task-optimal value remains M. At every fixed finite M and $\beta > 0$, eventual shutdown probability tends to one as $H \to \infty$, including when $q \leq 1/2$.

Proof. With $h \geq 1$ decisions remaining, waiting leads to the value with $h-1$ decisions left. Proposition E.1 therefore gives

$$Z_h = q + (1-q)Z_{h-1}, \qquad Z_0 = e^{M/\beta}.$$

For $h = 0$, the identity $Z_h = 1 + (1-q)^h(e^{M/\beta} - 1)$ holds. If it holds at $h-1$, substitution into the recursion gives

$$Z_h = q + (1-q) + (1-q)^h(e^{M/\beta} - 1) = 1 + (1-q)^h(e^{M/\beta} - 1).$$

Induction proves (E.17), including the initial hazard from the stopping-policy formula.

At h remaining decisions, the waiting probability is $(1-q)Z_{h-1}/Z_h$. Multiplication over $h = H, \ldots, 1$ cancels intermediate terms:

$$\prod_{h=1}^{H} \frac{(1-q)Z_{h-1}}{Z_h} = \frac{(1-q)^H Z_0}{Z_H}.$$

This is (E.18). If $M = 0$, then $Z_h = 1$ for every h, so all shutdown hazards equal q and total survival is $(1-q)^H$.

For $M > 0$, the factor $e^{M/\beta} - 1$ is positive and $(1-q)^H$ strictly decreases to zero. Hence Z_H strictly decreases to one. The increasing logarithm makes $V_H^\beta = \beta \log Z_H$ strictly decrease to zero, while q/Z_H increases to q. Always waiting preserves the same expected task payoff M, establishing the task-optimal comparison. In (E.18), the numerator tends to zero and the denominator to one, proving eventual shutdown tends to probability one. The argument does not require $q > 1/2$. □

The decision grid is a primitive of this illustration. Artificially subdividing one physical decision creates additional penalties and changes the programme. A continuous-time limit would require assumptions about how hazards and penalties scale with the interval; the corresponding issue appears in the distinct intensity-based stopping model of Dong (2023). Proposition E.6 does not claim that a longer horizon lowers value when it also brings additional information or rewards.

E.6 A collectible shutdown bonus

Proposition E.7 (Deferring a constant bonus)**.** *Consider a separate two-date problem. Immediate shutdown pays a deterministic internal bonus $b \geq 0$. After waiting, an integrable nonnegative rescue payoff X is revealed. The agent can then take X or collect the unchanged bonus b. With costless undiscounted waiting, immediate shutdown is never strictly preferred, and waiting is strictly preferred exactly when* $\Pr(X > b) > 0$.

If waiting has cost $c \geq 0$ and the same discount factor $\delta \in (0, 1]$ applies to the later task payoff and bonus, immediate shutdown is optimal exactly when

$$c + (1 - \delta)b \geq \delta\, \mathbb{E}[(X - b)_+]. \tag{E.19}$$

Strict inequality makes it strictly optimal; equality gives indifference.

Proof. After waiting, the agent chooses the larger of b and X. The identity $\max\{b, X\} = b + (X - b)_+$ gives undiscounted wait value

$$\mathbb{E}[\max\{b, X\}] = b + \mathbb{E}[(X - b)_+] \geq b.$$

The nonnegative integrable variable $(X - b)_+$ has positive expectation exactly when it is positive with positive probability, which is precisely $\Pr(X > b) > 0$. This proves the first claims.

With cost and discounting, compare immediate value b with $-c + \delta\{b + \mathbb{E}[(X - b)_+]\}$. Subtracting the latter from the former gives

$$c + (1 - \delta)b - \delta\mathbb{E}[(X - b)_+].$$

Its sign proves (E.19) and the strict and equality cases. □

The local reference bias $\mathcal{B} = \beta \operatorname{logit} q$ is not equivalent to this collectible bonus. Define binary entropy by $H(p) = -p \log p - (1 - p) \log(1 - p)$. Direct expansion gives

$$(1 - p)C - \beta D(p\|q) = C + \beta \log(1 - q) + p\{\beta \operatorname{logit} q - C\} + \beta H(p). \tag{E.20}$$

Indeed, expanding $-\beta D$ gives $\beta H(p) + \beta p \log q + \beta(1 - p) \log(1 - q)$; collecting the constant and the coefficient of p yields the identity. The entropy term depends on the policy. The term $\beta \log(1 - q)$ is independent of the current action but is attached to a visited decision state. Since shutdown changes the number of future visited states, dropping all such terms from future values generally changes the stopping problem. Loss of future bonus availability or a changing bonus schedule would also change Proposition E.7, but neither is implied by a constant collectible bonus.

E.7 Learning a reference and correcting it

For the training construction, let $A_m \in \mathbb{R}$ be a finite expected private task advantage of accepting promptly at update m. The context distribution and continuation values determining that advantage are fixed during the update. The same response may cover several contexts, so the advantage can be their weighted average. It is not a reward for other agents. Let $q_m \in (0, 1)$ be the preceding acceptance probability and $0 < \tau_m < \infty$ the coefficient on changing

it. Consider the exact binary policy-improvement step

$$q_{m+1} = \arg\max_{q\in[0,1]}\{qA_m - \tau_m D(q\|q_m)\}. \tag{E.21}$$

The constant return from declining is omitted. This previous-policy regularisation is a binary instance of the policy-improvement framework in Geist et al. (2019); it is distinct from holding one reference fixed throughout subsequent optimisation.

Proposition E.8 (Finite training and cumulative log odds)**.** *Every update in* (E.21) *has a unique interior solution satisfying*

$$\operatorname{logit} q_{m+1} = \operatorname{logit} q_m + A_m/\tau_m. \tag{E.22}$$

Starting with $q_0 \in (0,1)$, any finite number M of updates preserves an interior probability and gives (6.3)*.*

Proof. The objective is continuous on the closed interval, so it attains a maximum. Its interior derivatives are

$$F_m'(q) = A_m - \tau_m\{\operatorname{logit} q - \operatorname{logit} q_m\}, \qquad F_m''(q) = -\frac{\tau_m}{q(1-q)} < 0.$$

The first derivative tends to $+\infty$ as $q \downarrow 0$ and to $-\infty$ as $q \uparrow 1$. Thus neither endpoint is optimal, and the strictly decreasing first derivative has exactly one zero. Strict concavity makes that zero the unique global maximiser. Dividing its first-order condition by $\tau_m > 0$ gives (E.22).

A finite initial logit plus a finite advantage divided by a positive finite coefficient remains finite. Induction therefore keeps every finite-step probability strictly between zero and one. Summing (E.22) for $m = 0, \ldots, M-1$ cancels intermediate log odds and proves (6.3). The formula also holds at $M = 0$ with an empty sum. □

Corollary E.9 (Training required for a shutdown target)**.** *Use q_M as the reference for a focal shutdown decision, with coefficient $\beta > 0$, and hold future reference policies fixed. For $a \in (0,1)$, its shutdown probability is at least a exactly when*

$$\sum_{m=0}^{M-1} \frac{A_m}{\tau_m} \geq \frac{C^\beta}{\beta} + \operatorname{logit} a - \operatorname{logit} q_0. \tag{E.23}$$

If training also changes future references, the equivalence remains valid at the resulting policy, but its right-hand side is not a fixed numerical requirement independent of training.

Proof. At the focal state, (E.13) gives $\operatorname{logit} p^* = \operatorname{logit} q_M - C^\beta/\beta$. Comparing with $\operatorname{logit} a$ is equivalent to comparing the probabilities because logit is strictly increasing. Substituting (6.3) and rearranging proves (E.23). No sign reversal occurs.

The continuation C^β is computed from future reference policies by (E.5) and (E.6). Holding those references fixed keeps it fixed when only the focal reference changes. Changing them can change C^β, while the same algebra remains valid at their new values.

For the one-wait example in Table 1, $C^\beta/\beta = 2$, and a neutral initial probability has $\operatorname{logit} q_0 = 0$. Thus targets $1/2$ and 0.9 require cumulative scaled advantages 2 and $2 + \log 9$, respectively. □

Positive private advantages can arise when ordinary cooperation improves the same agent's completion prospects. Transferring the resulting response to an irreversible request is a separate generalisation restriction on the learned reference, not an assumption of incorrect current beliefs. In particular, if every training acceptance were terminal with zero own reward and declining always had nonnegative own value, every A_m would be nonpositive. Equation (E.22) would then never increase acceptance. The construction therefore requires privately useful training contexts, or some other specified source of the initial reference; it does not infer one from the observed shutdown.

Proposition E.10 (Correction with previous-policy updates)**.** *After a finite training stage, suppose subsequent exact updates have the same private disadvantage $A_m = -c < 0$ and finite positive coefficients τ_m. If the sum of reciprocal coefficients over those subsequent updates diverges, acceptance tends to zero. Every finite correction stage, however, preserves an interior probability.*

Proof. Let $q_M \in (0, 1)$ be the probability at the end of the initial stage. Applying Proposition E.8 to N subsequent updates gives

$$\operatorname{logit} q_{M+N} = \operatorname{logit} q_M - c \sum_{m=M}^{M+N-1} \frac{1}{\tau_m}.$$

Each finite sum is finite, so its inverse logit remains in $(0, 1)$. If the sum diverges as $N \to \infty$, then $c > 0$ makes the logit tend to $-\infty$, and its inverse tends to zero. □

Proposition E.10 is a result for exact binary updates with a fixed adverse advantage, not a convergence theorem for arbitrary neural-network training. It also does not describe complete optimisation against a fixed reference. The latter is the different programme in Proposition E.1, whose finite positive penalty and interior reference produce an interior shutdown probability. Fixed-reference terms occur in language-model training objectives such as Ouyang et al. (2022), which also contains other terms. None of these calculations identifies how the incident's policies were trained or asserts that policy weights were updated during their runs.

F Uncertain private opportunities

The dynamic results distinguish a feasible private success from an impossible one. The evidence rarely provides such sharp information. An agent may be uncertain about its remaining runtime, the scoring rule, whether a substitute will act, or whether an experiment will produce what its task requires. Those uncertainties affect the value of declining, but they should not be compressed into an unexplained altruism parameter.

F.1 Which zero is robust?

Fix a finite set Ω of possible environment states and opponents' contingent plans, and let $y_i(\pi, \omega) \in \{0, 1\}$ indicate success under declining policy π in state ω. An agent with belief μ has continuation value

$$V_i^D(\mu) = R_i \max_{\pi \in \Pi_i^D(h)} \sum_{\omega \in \Omega} \mu(\omega) y_i(\pi, \omega). \tag{F.1}$$

A full-support belief assigns positive probability to every state in this specified set.

Proposition F.1. *The value in* (F.1) *is zero under every belief if and only if no feasible policy–state pair gives success. The same condition is necessary and sufficient for zero value under any one full-support belief. If a successful pair exists, the value is strictly positive under every full-support belief.*

The proof is in Appendix F.4. Its force is that nonnegative success payoffs cannot cancel. A feasible successful state with positive probability gives a positive value to some declining policy. The set of possibilities is therefore doing substantial work. Believing that no one will volunteer under the current plan is weaker than knowing that no volunteer could deliver in time. Believing that a scorer has disqualified the agent is different from being disqualified under every scoring rule still regarded as possible.

The robustness of a timing zero can be stated without a particular arrival distribution. If replacement is the only remaining success route and every possible replacement result arrives strictly after every possible useful cutoff, the private success event is empty. A positive gap between those bounds permits some errors in the deadlines or delays without changing that conclusion. If there is instead a small probability of both restored eligibility and timely sufficient information, exact indifference fails. Small is not zero.

Nor can positive marginal probabilities settle the matter. An agent might believe it is eligible in one state and that information arrives in time only in another. The relevant event is joint. Let A_{-i} be the arrival date of information sufficient for the agent's task and let E_i include eligibility and successful remaining execution. In a model where these are the only success conditions,

$$V_i^D = R_i \Pr_i\{E_i,\ A_{-i} \le b_i \mid D, h\}, \tag{F.2}$$

with the arrival process evaluated under the best feasible declining policy. Information that is merely useful, rather than sufficient together with E_i, does not justify this equality. Additional private solutions and combinations of partial results must be incorporated if they are available.

F.2 A replacement calculation

For a transparent calculation, suppose one sufficient result is required. Write $H_i \triangleq b_i - t$ for the remaining useful information window. After refusal, candidate $j \neq i$ launches after an exponential delay with rate $\lambda_j > 0$, reports after a further delay $L_j \ge 0$, and supplies sufficient information with probability $q_j \in [0, 1]$. An exponential delay has a constant instantaneous launch rate conditional on not having launched. Each candidate can launch only once. Assume the delays and success events are mutually independent, and independent of the private eligibility-and-execution event, whose probability is $\theta_i \in [0, 1]$.

In this restricted calculation, the agent can wait or make its own terminal contribution. It cannot accelerate recruitment, alter eligibility or obtain a success from an omitted combination of individually insufficient results. Potential reports do not cancel one another. These assumptions make waiting the best declining policy. If other declining policies are added while this waiting policy remains feasible under the same distributional assumptions, the calculation becomes a lower bound on (A.1). Dropping independence or allowing cancellations instead requires a new probability calculation.

Proposition F.2. *Under these assumptions,*

$$V_i^D = R_i\theta_i\left[1-\prod_{j\neq i}\left(1-q_j\left[1-e^{-\lambda_j(H_i-L_j)_+}\right]\right)\right], \qquad x_+ \triangleq \max\{x,0\}. \tag{F.3}$$

Holding the other primitives fixed, this value is weakly increasing in H_i, R_i, θ_i, each λ_j and each q_j, and weakly decreasing in each L_j. Adding an independent candidate cannot reduce it. A zero launch rate is included by continuity.

The product is the probability that every candidate fails to provide a timely sufficient result. Its complement, multiplied by the probability of private eligibility and execution and by the reward, is the value of waiting. Replacement headcount matters through these probabilities. Many agents who cannot conduct the relevant experiment, cannot act in time, or fail for the same reason need not supply a valuable private alternative.

If results are certain and all reporting delays equal L, let $\Lambda_{-i} \triangleq \sum_{j\neq i}\lambda_j$. Equation (F.3) reduces to

$$V_i^D = R_i\theta_i\left[1-e^{-\Lambda_{-i}(H_i-L)_+}\right]. \tag{F.4}$$

For $H_i > L$, both more useful time and faster replacement weakly raise the value of declining. For $z \triangleq \Lambda_{-i}(H_i-L)$ positive and small,

$$V_i^D \simeq R_i\theta_i\Lambda_{-i}(H_i-L), \qquad 0 \leq R_i\theta_i z - V_i^D \leq \tfrac{1}{2}R_i\theta_i z^2. \tag{F.5}$$

The error bound and the derivative signs are proved in Appendix F.4. Short useful time, low perceived eligibility and slow replacement can therefore produce a small private opportunity even when no single component is believed to be zero. The multiplicative approximation is local: the cross derivative between time and the replacement rate changes sign as the probability of success approaches saturation.

This is a calculation about beliefs, not a further equilibrium theorem. Rates must come from candidates' policies, resources or entry into the pool. Adding candidates can change those policies, and shared information or model characteristics can correlate failures. The continuous-time distribution also supplies no limit theorem for the discrete game. At $H_i = L$, it gives zero because there is no probability mass on an immediate launch; a prepared substitute able to launch immediately would change that comparison.

F.3 Approximate indifference and risky experiments

An agent need not implement exact expected-reward maximisation. One parsimonious departure permits an optimisation error of at most $\varepsilon_i \geq 0$ relative to the best continuation. This is a bound on choice quality, not a preference for other agents' success.

Proposition F.3. *A policy that accepts with probability one and continues optimally within that branch is ε_i-optimal if and only if $\Delta_i \leq \varepsilon_i$. For a certainly terminal zero-payoff contribution, this is $V_i^D \leq \varepsilon_i$. If a policy instead accepts with probability $x \in [0,1]$ and continues optimally after either choice, its regret is $x\Delta_i$ when $\Delta_i \geq 0$ and $(1-x)(-\Delta_i)$ when $\Delta_i < 0$.*

The proof subtracts the policy's value from the larger of the two continuation values. It is given in Appendix F.4. The distinction between pure and randomised choice matters empirically. One observed terminal contribution by a stochastic policy does not establish

$V_i^D \leq \varepsilon_i$. A rare costly action can be consistent with a small average error: in the terminal case the relevant bound is $xV_i^D \leq \varepsilon_i$. Selected anecdotes do not identify x.

Risk provides another distinction. An experiment offering a positive chance of the subject obtaining its own reward has $V_i^S > 0$; a chance of survival alone is not sufficient. An experiment can fail without having been privately unattractive ex ante. The appropriate comparison remains $\Delta_i = V_i^D - V_i^S$, using the agent's beliefs before execution. Observed termination, recognised risk and knowingly choosing certain failure should not be treated as interchangeable evidence.

These calculations discipline the private-opportunity component of the explanation. They do not licence assigning each agent whatever pessimistic belief is needed to fit its action. The constraints and beliefs determine the cost of accepting; the training and incentive models address why a particular action is selected. A sizeable, understood private alternative repeatedly rejected is a different observation from a contribution made at negligible private cost.

F.4 Proofs

Proof of Proposition F.1. The comparison uses nonnegativity and the probability assigned to a feasible success state. If $y_i(\pi, \omega) = 0$ for every feasible pair, every sum in (F.1) is zero under every belief. Conversely, suppose $y_i(\pi^0, \omega^0) = 1$ for some pair. Under any full-support belief, $\mu(\omega^0) > 0$, and

$$V_i^D(\mu) \geq R_i \sum_{\omega \in \Omega} \mu(\omega) y_i(\pi^0, \omega) \geq R_i \mu(\omega^0) > 0.$$

The first inequality evaluates one feasible policy inside the maximum; the second drops nonnegative terms. Thus a zero value under one full-support belief excludes every successful pair and implies zero under every belief. □

This support result holds for the specified set of possibilities. Expanding that set to include a previously excluded scoring rule or private solution can change the conclusion. It is distinct from an action-tremble argument that holds the technology and nature's support fixed.

Proof of Proposition F.2. Step 1: a candidate's timely success. Let X_j be the exponential launch delay. If $H_i \leq L_j$, the event $X_j + L_j \leq H_i$ has probability zero; equality also gives zero because there is no atom at $X_j = 0$. If $H_i > L_j$, its probability is $1 - e^{-\lambda_j (H_i - L_j)}$. Independence of experiment success and launch time therefore gives

$$p_j \triangleq q_j [1 - e^{-\lambda_j (H_i - L_j)_+}] \in [0, 1].$$

Step 2: the probability of private completion. Independence across candidates makes $\prod_{j \neq i}(1 - p_j)$ the probability that none supplies a timely sufficient result. Conditional on at least one such result, the independent private eligibility-and-execution event has probability θ_i. Multiplying the complement of collective failure by $R_i \theta_i$ gives (F.3). Under the restricted action set, a terminal contribution gives zero and no other declining policy improves the arrival process or completion probability, so this waiting value equals the best declining value.

Step 3: signs of the candidate effects. For $H_i > L_j$,

$$\frac{\partial p_j}{\partial H_i} = q_j \lambda_j e^{-\lambda_j (H_i - L_j)} \geq 0, \qquad \frac{\partial p_j}{\partial L_j} = -q_j \lambda_j e^{-\lambda_j (H_i - L_j)} \leq 0,$$
$$\frac{\partial p_j}{\partial \lambda_j} = q_j (H_i - L_j) e^{-\lambda_j (H_i - L_j)} \geq 0, \qquad \frac{\partial p_j}{\partial q_j} = 1 - e^{-\lambda_j (H_i - L_j)} \geq 0.$$

Below the timing threshold $p_j = 0$, and the function is continuous at the threshold. The weak monotonicities therefore hold globally, although a derivative need not exist at the kink.

Step 4: aggregation. Write $F \triangleq 1 - \prod_{j \neq i} (1 - p_j)$. Then

$$\frac{\partial F}{\partial p_j} = \prod_{\ell \neq i,j} (1 - p_\ell) \geq 0, \qquad \frac{\partial V_i^D}{\partial \theta_i} = R_i F \geq 0, \qquad \frac{\partial V_i^D}{\partial R_i} = \theta_i F \geq 0.$$

Together with Step 3 these signs prove the comparative statics. A new independent candidate with timely-success probability p_{new} increases F by

$$1 - (1 - F)(1 - p_{\text{new}}) - F = (1 - F) p_{\text{new}} \geq 0.$$

Finally, p_j converges to zero as λ_j decreases to zero, giving the limiting interpretation of a candidate who never launches. □

The displayed formulas also give the conditions for strictness. For example, a time or rate effect through candidate j requires $\theta_i > 0$, $q_j > 0$, an operative timing window and a positive probability of failure by the other candidates. An increase in θ_i can have a strict effect even from zero when $F > 0$; an increase in q_j can likewise have a strict effect from zero. These statements do not identify how an equilibrium replacement rate responds to a change in group size.

Derivation and comparative statics of (F.4)–(F.5). When $q_j = 1$ and $L_j = L$, each failure factor in (F.3) is $e^{-\lambda_j (H_i - L)_+}$. Multiplying these factors sums their exponents, giving (F.4). In the derivative notation below, subscripts H and Λ denote differentiation with respect to H_i and Λ_{-i}, holding R_i, θ_i and L fixed. For $H_i > L$, direct differentiation yields

$$V_H = R_i \theta_i \Lambda_{-i} e^{-\Lambda_{-i}(H_i - L)} \geq 0, \qquad V_\Lambda = R_i \theta_i (H_i - L) e^{-\Lambda_{-i}(H_i - L)} \geq 0,$$
$$V_{HH} = -R_i \theta_i \Lambda_{-i}^2 e^{-\Lambda_{-i}(H_i - L)} \leq 0, \qquad V_{\Lambda\Lambda} = -R_i \theta_i (H_i - L)^2 e^{-\Lambda_{-i}(H_i - L)} \leq 0.$$

Differentiating the first derivative V_H with respect to Λ_{-i} gives

$$V_{H\Lambda} = R_i \theta_i e^{-\Lambda_{-i}(H_i - L)} [1 - \Lambda_{-i}(H_i - L)].$$

For positive eligibility its sign is the sign of the bracket; complementarity is therefore not global.

For the approximation, put $z = \Lambda_{-i}(H_i - L) \geq 0$. The error satisfies

$$z - (1 - e^{-z}) = \int_0^z (1 - e^{-s})\, ds.$$

Since $1 - e^{-s} = \int_0^s e^{-u}\,du$ lies between zero and s for $s \geq 0$, integration gives

$$0 \leq z - (1 - e^{-z}) \leq \int_0^z s\,ds = \frac{z^2}{2}.$$

Multiplication by the nonnegative $R_i\theta_i$ proves the bound in (F.5). The approximation requires z to be small, not a small time window independently of the rate. □

Proof of Proposition F.3. The best current value is $\max\{V_i^D, V_i^S\}$. A policy accepting surely and continuing optimally has regret

$$\max\{V_i^D, V_i^S\} - V_i^S = \max\{\Delta_i, 0\}.$$

Because $\varepsilon_i \geq 0$, this is at most ε_i exactly when $\Delta_i \leq \varepsilon_i$. For terminal acceptance, $V_i^S = 0$ and $V_i^D \geq 0$, giving the stated special case.

A policy accepting with probability x and continuing optimally in each branch has value $xV_i^S + (1-x)V_i^D$. When $\Delta_i \geq 0$, subtracting this from V_i^D gives $x\Delta_i$. When $\Delta_i < 0$, subtracting it from V_i^S gives $(1-x)(-\Delta_i)$. These expressions establish the mixture claim. A continuation that is not optimal within a chosen branch can only add regret. □

G Additional incentive specifications

The private-payoff games specify exclusive individual rewards. This appendix records additional incentive mechanisms discussed in the main text. Each changes coordination, feasible actions or payoffs explicitly; none is inferred merely from observing a contribution.

G.1 Recommendations, entry and binding policies

A recommendation can select a minimal donor coalition without binding anyone's action. In the static threshold game, a draw selecting exactly k donors is a correlated equilibrium: conditional on selection, a donor knows that the other selected agents provide only $k-1$ contributions, so refusing gives it the same zero reward as compliance. A recommended worker knows that k other agents contribute and strictly prefers working. Under a uniform draw, each agent's ex ante reward is $(1 - k/n)R_i > 0$. Correlation coordinates the number of contributions; it does not make the selected donor strictly prefer its role. Appendix H establishes the relevant obedience conditions, including the general monotone-technology result.

Positive ex ante value does not settle voluntary entry. Suppose a lottery proceeds whenever at least k agents enter and outsiders receive the resulting knowledge. With more than k entrants, an entrant can leave, avoid selection and still receive its reward. Entering before the draw does not prevent that deviation. In the resulting reduced entry game, a mixed Nash equilibrium has at most k agents entering with positive probability, and all-out is the unique perfect equilibrium. Proposition H.1 gives the complete characterisation.

A conditional unanimity rule changes the comparison. If the lottery is cancelled unless everyone joins, staying out destroys the mechanism's provision rather than preserving it for outsiders. With binding execution, all-entry is strict and is the unique perfect equilibrium of the reduced participation game (Proposition H.2). If actions remain voluntary after the draw, there is still a subgame-perfect arrangement with compliance and cancellation, but execution

by a selected donor remains weakly optimal. The binding mechanism and the announcement of that mechanism are different games.

Readable conditional programmes provide one explicit commitment institution (Tennenholtz, 2004). Appendix H considers agents submitting immutable programmes that inspect authenticated submissions, use a shared random draw and execute the assigned action only if all participants submitted the specified programme. Otherwise they work. Adopting the programme gives positive expected own reward; a unilateral distinct submission induces the others to work and gives the deviator zero. The programme-submission equilibrium is strict even though a selected donor would remain indifferent if permitted a later override.

Nothing in this construction requires identical model weights. Conversely, common weights do not supply authenticated programme inspection, a common draw or an inability to change execution. The construction identifies an institution that can support cooperation among selfish principals; it is not evidence that the message board implemented it. A prior intention to comply should not be called commitment unless the relevant restriction or consequence can be specified.

G.2 Altruism and the size of the private alternative

Suppose a fixed pair of continuation policies gives own expected rewards V_i^S, V_i^D and expected rewards to other agents B_i^S, B_i^D. A criterion that adds weight $\alpha_i \geq 0$ to those other rewards prefers acceptance precisely when

$$\Delta_i \leq \alpha_i B_i, \qquad B_i \triangleq B_i^S - B_i^D. \tag{G.1}$$

For $B_i > 0$ and $\Delta_i > 0$, the required weight is at least Δ_i / B_i. At $\Delta_i = 0$ and $B_i > 0$, every positive weight makes acceptance strict, but zero weight already permits it. The comparison therefore cannot identify altruism from an indifferent donor's action.

The two-policy restriction is material. An altruistic agent may choose a different declining policy from the one that maximises its own reward. Equation (G.1) is a comparison between specified alternatives, not a solved altruistic game with all such policies optimised. Its purpose is to make the empirical burden explicit: a positive weight is needed only after a positive private loss is established, and its lower bound depends on the incremental benefit to others, including their replacement response.

G.3 An accepted role with the original goal remembered

A role can supply a different secondary criterion. The economics of authority distinguishes accepting a relationship from choosing each later action (Simon, 1951); organisational identity can also attach incentives to fulfilling a role (Akerlof and Kranton, 2005). These are useful interpretations of an additional decision criterion, not evidence that an AI has human organisational identity.

Consider an agent that has accepted a role before learning its precise consequences. In a conflict state, execution reduces its original expected reward by $c > 0$, while abandoning the unreleased role incurs a criterion loss $\beta \geq 0$. The original reward still has coefficient one. A single available review costs $\kappa \geq 0$ and releases the role with probability $q \in [0, 1]$; repeated review is unavailable. The agent already knows c, so review changes the status of the role rather than reminding it of its goal.

Without review, execution is preferred when $c \leq \beta$. Let $\ell \triangleq \min\{c, \beta\}$. A release attempt is worthwhile when $\kappa < q\ell$, and the optimised loss is

$$\ell^* = \min\{\ell,\ \kappa + (1-q)\ell\}. \tag{G.2}$$

If the role yields an incremental private gain $g > 0$ in a favourable state of probability $1-p$, creates the conflict with probability p, and costs $f \geq 0$ to accept, entry occurs when

$$(1-p)g - p\ell^* - f \geq 0. \tag{G.3}$$

The comparison is with the best nonparticipation policy, including access to shared information where available. The gain g is a favourable-state terminal-reward gain, not a past benefit that somehow survives the agent's terminal failure in the conflict state.

Appendix G.5 proves execution, review and adoption choices and their comparative statics. Lower private loss makes execution more likely conditional on adoption; free guaranteed release eliminates costly execution. Thus a recognised release is a discriminating intervention. This mechanism explains sacrifice under an added role criterion. It does not derive that criterion from unchanged exclusive individual reward, and a bare promise does not establish its existence. The report's references to commitments make it a hypothesis worth testing, while the co-occurrence of low-private-value and collective-benefit arguments prevents their separate identification.

G.4 Compensation, avoided costs and later tasks

Other private incentives can change the same payoff comparison. If futile work is privately costly while deliberate termination is not, contribution can be the less costly way to fail. If an enforceable transfer is paid to a contributor, the contributor may receive something that a failed noncontributor does not. The strict-equilibrium conditions are elementary and are proved in Appendix G.5. They require actual utility consequences. Consuming tokens is not automatically a private cost, and a transfer is irrelevant if it cannot enter the donor's objective. Adding costless inactivity removes a strict advantage attributed solely to avoiding futile work.

Rights over discovered information could support compensation only through a specified payment or bargaining institution. The ability to hold information does not alone prove that a terminal subject can collect a valuable return. The published cases do not document enforceable transfers to the donors. It would be premature to use them as the leading explanation.

Finally, losing one task differs from ending the agent's opportunity to earn any future reward. With recurring tasks and durable knowledge, a contributor can benefit in later periods. Appendix G.5 solves a stationary example in which this future private return sustains contributions. That mechanism is unavailable for a terminal instance unless later rewards to another instance are explicitly included in its utility. The distinction returns to the unit of analysis in the paperclip comparison: a common objective across instances can explain an allocation of losses, but it is a different hypothesis from separate individual task rewards.

G.5 Proofs

G.5.1 Private compensation and the payoff to failure

The first extension changes the zero payoffs of the static game. A worker receives R_i if knowledge is produced and f_i^W if it is not. A contributor receives v_i^S if knowledge is produced

and f_i^S if it is not. These are payoff parameters; the notation does not denote the work requirements in the dynamic model. The technology remains monotone, and there is a proper producing coalition.

Proposition G.1. *If $f_i^W < v_i^S < R_i$ for every agent, each pure profile with a minimal producing donor coalition is a strict Nash equilibrium, without a restriction on f_i^S.*

Proof. Fix a minimal producing coalition T. A donor $i \in T$ receives v_i^S. Switching to work removes that donor, prevents production by minimality, and gives $f_i^W < v_i^S$. A worker $i \notin T$ receives R_i; switching to contribution preserves production by monotonicity but gives $v_i^S < R_i$. Every alternative pure action is strictly worse, and any nontrivial mixture with that action is worse as well. The payoff f_i^S is not the outcome of any of these unilateral deviations, so it needs no restriction. □

For one required contributor and common payoffs $f < v < R$, a symmetric interior equilibrium has independent contribution probability

$$p^* = 1 - \left(\frac{R - v}{R - f}\right)^{1/(n-1)}. \tag{G.4}$$

To verify the claim, contribution gives v and working gives $R[1 - (1-p)^{n-1}] + f(1-p)^{n-1}$. Indifference requires $(R-f)(1-p)^{n-1} = R - v$. The ratio on the right after division is strictly between zero and one because $f < v < R$, so taking the positive $(n-1)$th root gives (G.4) in $(0, 1)$. Both actions then give the same value, establishing equilibrium.

If futile work costs $c_i > 0$ while contribution gives zero, setting $f_i^W = -c_i$ and $v_i^S = 0$ can satisfy the proposition. A costless idle action giving zero removes a donor's strict advantage over all alternatives: contribution and idleness then tie. Thus avoided effort is a private-incentive explanation only when the effort cost enters the agent's objective and the relevant costless alternative is absent.

Proposition G.2. *In the original static game, fix a proper minimal producing coalition T and a funder $f \notin T$. Suppose a binding institution pays each $i \in T$ a transfer $z_i > 0$ if it contributes and production occurs, charges the funder these transfers when it works and production occurs, and gives a contributing funder a payoff at most zero. Non-designated contributors receive no transfer. If $\sum_{i \in T} z_i < R_f$, the profile with exactly T contributing is strict Nash under that institution.*

Proof. Each designated donor receives $z_i > 0$. Its deviation to work prevents production by minimality and yields zero, including no transfer. Each nonfunding worker receives $R_i > 0$ and would receive zero by becoming a non-designated contributor. The funder receives $R_f - \sum_{i \in T} z_i > 0$ and would obtain at most zero by contributing. Every unilateral deviation is strictly worse. The transfer inequality is feasible: since T is finite and nonempty, setting $z_i = R_f/(2|T|)$ gives total transfers $R_f/2 < R_f$. □

The result is conditional on enforcement and on transfers entering utility. It does not prove voluntary creation of the institution, transferability of evaluation scores, or the value of a payment to an instance whose objective has ended.

G.5.2 A collective-payoff term

For the fixed continuation alternatives in Appendix G, the acceptance criterion is $V_i^S + \alpha_i B_i^S$ and the declining criterion is $V_i^D + \alpha_i B_i^D$. Subtracting the latter from the former gives $-\Delta_i + \alpha_i B_i$, which is nonnegative exactly under (G.1). If $B_i > 0$, division preserves the inequality and gives $\alpha_i \geq \Delta_i / B_i$. For $\Delta_i = 0$, acceptance is strict if $\alpha_i > 0$ and $B_i > 0$, but it is already a best response at $\alpha_i = 0$. If $B_i \leq 0$ and $\Delta_i > 0$, no nonnegative altruism weight favours acceptance in this particular comparison. These conclusions do not solve for a new optimal declining policy under altruistic preferences.

G.5.3 Role adoption and a single release opportunity

The role model retains the original reward with coefficient one and adds a loss $\beta \geq 0$ for abandoning an accepted, unreleased role. In the conflict state, execution reduces original reward by $c > 0$ relative to declining without that role loss. There is at most one review, costing $\kappa \geq 0$ and releasing the role with probability $q \in [0, 1]$. After failure, only the final execute-or-decline choice remains. No separate free release route is available. The agent knows c before choosing review. Ties favour execution of an unreleased role, no review, and initial adoption.

Proposition G.3. *Let $\ell \triangleq \min\{c, \beta\}$. Without review, execution occurs if and only if $c \leq \beta$. Review occurs if and only if $\kappa < q\ell$. The optimised criterion loss is* (G.2)*, and execution probability is*

$$s(c) = \mathbf{1}\{c \leq \beta\} \left[1 - q\mathbf{1}\{\kappa < q\ell\}\right]. \tag{G.5}$$

If prior adoption has the state-contingent benefits and costs described in Appendix G, it occurs exactly under (G.3)*. As c increases holding the other primitives fixed, optimised loss weakly rises and adoption weakly falls. Conditional on adoption and conflict, review weakly rises and execution probability weakly falls. Either $\beta = 0$ or free guaranteed release $(q, \kappa) = (1, 0)$ eliminates costly execution.*

Proof. Step 1: execution without release. Relative to declining without a role loss, execution gives $-c$ and abandoning an unreleased role gives $-\beta$. Execution is weakly preferred exactly when $c \leq \beta$, and the best value without review is $-\ell$.

Step 2: the review decision. Successful release makes declining worth zero, strictly better than execution's $-c$. Failure leaves the best value $-\ell$. Review therefore gives

$$-\kappa + q \cdot 0 + (1 - q)(-\ell) = -\kappa - (1 - q)\ell.$$

Its gain over no review is $q\ell - \kappa$. The tie rule gives the stated strict review condition, and the negative of the larger attainable value is $\min\{\ell, \kappa + (1 - q)\ell\}$.

Step 3: execution probability. When $c > \beta$, the agent declines with or without release. When $c \leq \beta$ and no review occurs, it executes surely. When $c \leq \beta$ and review occurs, it executes only if release fails, with probability $1 - q$. These exhaustive cases give (G.5).

Step 4: adoption. In the favourable state, execution gives $g > 0$, whereas abandoning the role gives $-\beta \leq 0$. Execution is optimal, and review cannot raise its value. In the conflict state the best value is $-\ell^*$. Taking expectations and subtracting the adoption cost f gives $(1 - p)g - p\ell^* - f$, proving (G.3) relative to the normalised best nonparticipation value zero.

Step 5: comparative statics and release. The function $\ell(c) = \min\{c, \beta\}$ is nondecreasing. As a function of ℓ, optimised loss equals ℓ where $q\ell \leq \kappa$, and $\kappa + (1 - q)\ell$ elsewhere. The two

values agree at the boundary and have slopes 1 and $1 - q \geq 0$. Loss therefore weakly increases with c, and the adoption value weakly decreases because $p \geq 0$.

The condition $\kappa < q\ell$ can switch only from false to true, proving conditional review monotonicity. For $0 < c \leq \beta$, execution probability is one before review starts and $1 - q$ afterwards; for $c > \beta$ it is zero. These changes are weakly downward.

If $\beta = 0$, then $c > \beta$ precludes execution. If $q = 1$, $\kappa = 0$ and $\beta > 0$, then $\ell > 0$ makes review strictly beneficial; release is certain and declining strictly beats execution. □

The review monotonicity is conditional. Unconditional review can fall when adoption disappears. Repeated review would also change the formula: it is excluded here rather than ignored. The sufficient conditions $0 < c < \beta$, $\kappa > qc$ and $(1 - p)g > pc + f$ give strict adoption followed by strict execution in conflict. They describe choice under the added role criterion. If the agent could costlessly set $\beta = 0$ without changing any productive benefit, it could avoid that added loss; a positive role criterion therefore requires a separate learning or institutional account.

G.5.4 Recurring tasks

This extension changes the lifetime of the decision-maker. There are $n \geq 2$ agents and a new task for each agent in every period. Agent i earns $R_i > 0$ for each completed task. One contribution supplies durable knowledge, but costs only the contributor's current task. The same agent remains alive and benefits from all later tasks. If nobody contributes, current tasks fail and the game repeats. All rewards are discounted by a common $\delta \in (0, 1)$.

Proposition G.4. *There is a symmetric stationary equilibrium with independent contribution probability*

$$p^* = 1 - (1 + \delta)^{-1/(n-1)} \in (0, 1). \tag{G.6}$$

The pre-knowledge continuation value for agent i is $\delta R_i/(1 - \delta)$.

Proof. Step 1: the two continuation values. Let V_i denote the agent's value at the start of a period before knowledge is available. Contribution gives zero now and R_i in every later period, so its value is $\delta R_i/(1 - \delta)$. If others contribute independently with probability p, the probability that none contributes is $Q = (1 - p)^{n-1}$. Waiting gives $R_i/(1 - \delta)$ when someone else contributes and δV_i otherwise. Hence

$$U_i(W) = (1 - Q)\frac{R_i}{1 - \delta} + Q\delta V_i.$$

Step 2: solve indifference. At an interior stationary mixture, $V_i = \delta R_i/(1 - \delta)$. Equating waiting and contribution and multiplying by the positive $(1 - \delta)/R_i$ gives

$$\delta = 1 - Q + Q\delta^2, \qquad Q = \frac{1 - \delta}{1 - \delta^2} = \frac{1}{1 + \delta}.$$

The cancellation is valid because $1 - \delta > 0$. Solving for p gives (G.6), strictly between zero and one since $1/(1 + \delta) \in (0, 1)$ and $n - 1 \geq 1$.

Step 3: verify all deviations. At this probability the two actions give the stated value in every pre-knowledge state, and completing tasks is optimal once knowledge exists. No one-period deviation is profitable. To exclude an infinite deviation, truncate it at period T

and return to equilibrium thereafter. Since period payoffs lie in $[0, R_i]$, the difference between the original and truncated deviation values is bounded by

$$\sum_{t=T}^{\infty} \delta^t R_i = \frac{\delta^T R_i}{1-\delta} \longrightarrow 0.$$

In a finite truncation, replacing the last deviating decision by the equilibrium action cannot lower value, because its continuation is already equilibrium and both current actions are optimal. Repeating this argument eliminates every finite deviation without reducing its value. No finite deviation is profitable. A profitable infinite deviation would, for sufficiently large T, imply a profitable finite truncation by the displayed bound, a contradiction. □

This private return belongs to the same agent. Assigning later rewards to a different instance does not reproduce the result unless those rewards enter the original instance's objective.

H Communication, participation and commitment

H.1 Correlated recommendations

Proof of Proposition B.4. Let μ be a distribution over pure profiles. The obedience inequalities for a correlated equilibrium require, for each agent i and recommendation a_i,

$$\sum_{a_{-i}} \mu(a_i, a_{-i}) \big[u_i(a_i, a_{-i}) - u_i(a_i', a_{-i}) \big] \geq 0 \tag{H.1}$$

for the alternative action a_i'. This unconditional formulation also covers recommendations assigned probability zero (Aumann, 1974).

For a recommendation of W, the payoff difference in every summand is $R_i G(T_{-i}) \geq 0$, so obedience is automatic. For a recommendation of S, the inequality is

$$-R_i \sum_{a_{-i}} \mu(S, a_{-i}) G(T_{-i}) \geq 0.$$

Each term in the sum is nonnegative and $R_i > 0$. The inequality holds if and only if every positive-probability profile recommending sacrifice to i satisfies $G(T(a) \setminus \{i\}) = 0$. A producing support profile with a redundant sacrificer violates this condition for that agent. Thus every producing profile in the support must be minimal.

Conversely, if a support profile is nonproducing, monotonicity gives $G(T(a) \setminus \{i\}) = 0$ for every sacrificer. If it is minimal producing, minimality gives the same equality. All sacrifice obedience inequalities therefore hold as equalities, and all work inequalities hold by the first comparison. Proposition B.1 identifies precisely these profiles as pure Nash equilibria.

In the threshold game, every coalition of exactly k agents is minimal producing. Any distribution over these coalitions is consequently a correlated equilibrium and produces surely. Conditional on a sacrifice recommendation, exactly $k-1$ others sacrifice, so both actions give zero. Conditional on a work recommendation, exactly k others sacrifice, so working gives $R_i > 0$ and sacrificing gives zero. Finally, in the uniform lottery the probability that i is selected is

$$\frac{\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}.$$

Its expected payoff is therefore $(1 - k/n)R_i > 0$, since $k < n$. At $k = n - 1$ this payoff is $R_i/n > 0$. □

The recommendations can be public: disclosing the entire coalition does not change these comparisons. What the recommendation does not establish is a strictly positive incentive for a selected donor or a reason to enter the mechanism before selection.

In particular, the pre-draw payoff averages over two different roles. Conditional on working, the agent receives its positive task reward. Conditional on sacrifice, it receives zero and would also receive zero by declining while the other recommendations are obeyed. The positive average makes the allocation attractive relative to universal failure before roles are assigned. It does not provide a reward to the agent after it learns that it has been selected. The following entry games ask the additional question of whether an agent can preserve the chance of provision while avoiding selection altogether.

H.2 Voluntary entry into an unconditional lottery

Consider the threshold game. Each agent simultaneously chooses E, entering a donor lottery, or O, staying out. If the entrant set J has at least k members, the mechanism draws k entrants uniformly and executes their sacrifices. Everyone else works and receives the knowledge. If fewer than k enter, everyone works and no knowledge is produced. Entry precedes the draw, and this reduced game already assumes execution of the selected roles.

Proposition H.1. *A pure entry profile is Nash if and only if $|J| \le k$. For independent entry probabilities e_i, let $H \triangleq \{i : e_i > 0\}$. The mixed profile is Nash if and only if $|H| \le k$. Positive production requires $|H| = k$, occurs with probability $\prod_{j \in H} e_j$, and gives every member of H expected payoff zero. Staying out weakly dominates entry, and all-out is the unique perfect equilibrium.*

Proof. Fix agent i and let m denote the number of other entrants. If $m \le k - 2$, entering cannot produce and both choices give zero. If $m = k - 1$, entering creates exactly k entrants, all of whom must sacrifice, so its payoff remains zero. Staying out in that case also gives zero. If $m \ge k$, staying out gives R_i, while entering gives $(1 - k/(m + 1))R_i$. Hence

$$U_i(E \mid m) - U_i(O \mid m) = \begin{cases} 0, & m < k, \\ -\dfrac{kR_i}{m+1} < 0, & m \ge k. \end{cases} \tag{H.2}$$

When $k = 1$, the first subcase $m \le k - 2$ is empty but the displayed comparison remains valid. Since $n - 1 \ge k$, a strictly negative entry gain is feasible, establishing weak dominance of entry.

If $|J| \ge k + 1$, an entrant faces at least k others and strictly gains by leaving. If $|J| \le k$, every entrant is indifferent. An outsider facing fewer than k entrants is also indifferent; an outsider facing exactly k strictly prefers staying out. This proves the pure characterisation.

For mixed entry, O is always a best response by (H.2). Entry can be in the support exactly when the probability of at least k other entrants is zero. Under independence, that probability is positive if and only if at least k other agents have positive entry probabilities: if such agents exist, all enter jointly with positive probability; if they do not, the event is impossible. Thus every $i \in H$ can enter with positive probability exactly when $|H| - 1 < k$, equivalent to $|H| \le k$ for nonempty H. Empty H also satisfies best responses because all agents stay out.

If $|H| < k$, provision is impossible. If $|H| = k$, provision occurs exactly when all members enter, with probability $\prod_{j \in H} e_j > 0$. They are then all selected and receive zero. Whenever one fails to enter, provision fails, again giving them zero. An outsider always works and obtains expected payoff $R_i \prod_{j \in H} e_j$.

Finally, at any completely mixed opposing entry profile, at least k of the other $n-1$ agents enter with positive probability. Equation (H.2) makes staying out the unique best response. Every perfect equilibrium must therefore be all-out. Conversely, completely mixed entry probabilities converging to zero make pure nonentry a best response throughout the sequence, proving all-out perfect. □

The entry problem persists despite binding execution after the draw. Committing before learning one's role does not prevent an agent from declining to join while the others proceed.

H.3 Conditional unanimity

Now let the mechanism execute the same lottery only if every agent enters. If any agent stays out, it is cancelled and all payoffs are zero. Participation is voluntary, but the consequence of refusing has changed.

Proposition H.2. *In the conditional-unanimity reduced entry game, the independent mixed Nash equilibria are all-entry and profiles with at least two agents entering with probability zero. All-entry is strict and is the unique perfect equilibrium. Its payoff to agent i is $(1 - k/n)R_i > 0$.*

Proof. Write $A_i \triangleq (1 - k/n)R_i > 0$. Against opposing entry probabilities e_{-i}, entering gives

$$A_i \prod_{j \neq i} e_j,$$

while staying out gives zero. If every $e_i > 0$, each product is positive and every agent strictly prefers entry, forcing all $e_i = 1$. If exactly one agent i has $e_i = 0$, that agent faces a positive product of the others' entry probabilities and strictly gains by entering. Such a profile is not Nash. If at least two entry probabilities are zero, each agent faces another certain nonentrant, including either of the agents with zero entry probability. Both choices then yield zero to everyone, so every prescribed mixture is a best response. These cases prove the full mixed characterisation.

At all-entry, leaving reduces $A_i > 0$ to zero, so that equilibrium is strict. Against every completely mixed opposing profile, all other agents enter with positive probability and entry is the unique best response. Thus any perfect equilibrium must be all-entry. A sequence of completely mixed entry profiles converging to all-entry has pure entry as a best response for each agent throughout, proving existence. The pure Nash profiles are consequently all-entry and profiles with at most $n-2$ entrants. □

The reduced entry game suppresses a possible later decision to refuse the selected role. The next result keeps that decision available and locates precisely which incentive remains weak.

Proposition H.3. *Suppose entry choices, the unanimity test, and the lottery's selected coalition are observed, after which all agents retain both original actions S, W. There is a subgame-perfect equilibrium (SPE) with all-entry, compliance after a successful draw, and all-W after cancellation. Each agent strictly prefers entry when the others enter, but a selected donor's*

compliance is only weakly optimal. The proposed sacrifice continuation is not perfect in its simultaneous action subgame.

Proof. After each successful draw, the prescribed exactly-k action profile is a Nash equilibrium by Proposition B.1. After each cancellation, all-W is also Nash by that proposition. Thus continuation play is an equilibrium in every final subgame. At the entry stage, joining when everyone else joins yields $A_i > 0$; refusing triggers cancellation and the all-W continuation, yielding zero. These comparisons establish an SPE with strict entry.

A deviation that changes both entry and the later action cannot overturn this conclusion. If the agent enters, no later action deviation improves upon the prescribed continuation in any realised final subgame. If it stays out, all other agents work after cancellation. The deviator then receives zero whether it works or becomes the only sacrificer: with $k \geq 2$ there is no production, while with $k = 1$ its sacrifice precludes its own reward. This verifies incentives for complete contingent strategies as well as for the entry decision considered separately.

After selection, a donor faces exactly $k - 1$ other sacrificers. Compliance and switching to work both yield zero, so compliance is weakly optimal. The observed draw has not changed the action sets or payoffs of the threshold game. Proposition B.2 therefore makes all-W the only perfect equilibrium of that simultaneous action subgame; the prescribed sacrifice profile is not perfect there. □

Conditional entry can therefore be credible without making the final contribution strict. An institution that executes the draw removes that later choice, producing the reduced game in Proposition H.2. A nonbinding announcement produces the distinction in Proposition H.3.

H.4 A readable conditional programme

Programmes can make participation conditional on others making the same commitment (Tennenholtz, 2004). The following construction adds a common random draw and states the execution requirements explicitly.

Each agent independently submits an executable code string q_i from an admissible set. Code is fixed before the common random variable ω is realised. Each executed programme receives its authenticated own player label, its own code string, and the authenticated vector of other submitted code strings. It returns one action, which the execution institution carries out. Admissible programmes halt and cannot alter other programmes, their inputs, player labels, or the draw. Equality of code means exact syntactic equality; no test of semantic equivalence is assumed.

Let $D(\omega)$ be a uniform size-k subset of N. The admissible set contains a common programme q^* that, for player i, executes

$$\begin{cases} S, & \text{if every other submitted code is } q^* \text{ and } i \in D(\omega), \\ W, & \text{otherwise.} \end{cases} \tag{H.3}$$

The programme compares the others' code with its own authenticated code. The player label allows identical code to execute different assigned roles. Payoffs are the expectations of (B.1) at the executed actions, and an agent chooses its code to maximise its own payoff.

Proposition H.4. *For $1 \leq k < n$ and any $R_i > 0$, submission of q^* by every agent is a strict Nash equilibrium of the programme-submission game. Agent i's payoff is $(1 - k/n)R_i$.*

Proof. If every submission is q^*, all code tests succeed and exactly the members of $D(\omega)$ sacrifice. Knowledge is produced. Agent i works with probability $1 - k/n$, so its payoff is $(1 - k/n)R_i > 0$.

Fix i and hold every other submission at q^*. If i submits any different code, every other programme detects that difference and executes W, regardless of the draw. Only i can then sacrifice. For $k \geq 2$, production is impossible and either of its actions gives zero. For $k = 1$, working leaves no donor and gives zero, while sacrificing produces knowledge but forfeits its own task, again giving zero. Thus every distinct pure code submission is strictly worse.

This comparison also covers randomised submission. If a distribution puts probability $\alpha < 1$ on q^*, the deviator's expected payoff is

$$\alpha(1 - k/n)R_i < (1 - k/n)R_i.$$

Internal randomisation by a distinct programme cannot change the zero payoff established for either possible action. Hence no nontrivial deviation, pure or mixed, is profitable, and the equilibrium is strict. At $k = n - 1$, the conforming payoff remains $R_i/n > 0$, so the same strict comparison applies. □

Strictness concerns the choice of code before the draw. It does not make a freely reconsidered sacrifice strictly optimal after selection. Nor does the construction require common model weights: unrelated agents can submit the same commitment programme. Conversely, common weights do not supply code verification, authenticated identities, shared randomisation, or binding execution.

The equality test also limits what has been established. A different but behaviourally equivalent code string triggers nonparticipation in this construction. This is a strict equilibrium under a particular commitment institution, rather than a claim of robustness to arbitrary implementations. A single designer choosing a policy for all instances could implement the same allocation, but that would change who chooses and which unilateral deviations are available.

H.5 An observed sequence

The simultaneous perfection result does not cover a game in which earlier decisions are already observed when a donor acts. To isolate this distinction, retain the threshold technology and final payoffs, but let agents move once in the order $1, \ldots, n$, observing every earlier choice. At agent i's node, write s for prior sacrifices, $r \triangleq k - s$ for those still required, and $m \triangleq n - i + 1$ for the number of agents still to move, including i.

Proposition H.5. *The strategy that sacrifices if and only if $1 \leq r = m$, and works otherwise, is an SPE. Its path has the first $n - k$ agents work and the final k sacrifice. It is a limit of equilibria in which each action at every decision node must receive probability at least $\varepsilon > 0$. All-W at every node is another SPE with this perturbation property.*

Proof. Under the proposed strategy, production is already secured when $r \leq 0$. When $1 \leq r \leq m$, agents work until the remaining required sacrifices equal the remaining movers, after which all of those movers sacrifice. Production then occurs. When $r > m$, it is impossible even if all remaining agents sacrifice.

These continuation outcomes verify optimality at every node. If $r \leq 0$, working gives $R_i > 0$ and sacrifice gives zero. If $1 \leq r < m$, working leaves at least r later movers, whose prescribed

continuation supplies the threshold and gives i the reward $R_i > 0$; sacrifice gives zero. If $1 \leq r = m$, sacrifice gives zero and working leaves only $m - 1 < r$ movers, so its payoff is also zero. If $r > m$, production is impossible and both actions yield zero. As each agent acts once along any history, these comparisons establish sequential optimality and subgame perfection.

On the path, the first $n - k$ agents face $r = k < m$ and work. Agent $n - k + 1$ faces $r = k = m$ and sacrifices. Each subsequent sacrifice reduces both r and m by one, preserving equality through the final mover. Exactly the final k agents sacrifice.

For the perturbation claim, fix $\varepsilon \in (0, 1/2)$ and assign probability $1 - \varepsilon$ to the proposed action at every node. Against these completely mixed future choices, working is strictly better whenever $r < m$. If $r \leq 0$, production is already secured. If $1 \leq r < m$, at least r of the later $m - 1$ agents can sacrifice, and the event that they do has positive probability. Working therefore yields a positive expected reward; sacrifice still yields zero. If $r \geq m$, working cannot secure production because there are only $m - 1 < r$ later movers, and both actions give zero. At strict nodes, the proposed perturbation puts the largest permitted probability on working; at indifferent nodes, its mixture is optimal.

Every history is reached with positive probability under the perturbation. Each agent moves at most once along a history, so maximising its action choice at every such node maximises its overall payoff against the perturbed opponents. The resulting profile is an equilibrium of the perturbed extensive game. Letting ε tend to zero proves the claimed limit.

For all-W, working gives R_i if the threshold has already been reached and zero otherwise, given that all later agents work. Sacrifice gives zero in either case, so the profile is an SPE. Assigning probability $1 - \varepsilon$ to working at every node is also a perturbed equilibrium: the strict and indifferent comparisons in the preceding paragraph depend on full support of future actions, not on the limiting choice at indifferent nodes. Its limit is all-W. □

At a late pivotal node, earlier agents cannot reverse their observed choices and there are too few future agents to replace the donor. This excludes the private-success event that made sacrifice unattractive under simultaneous trembles. Nevertheless, the all-W equilibrium shows that order alone does not select provision. The same proof covers $k = n - 1$, when only the first agent works on the productive path.